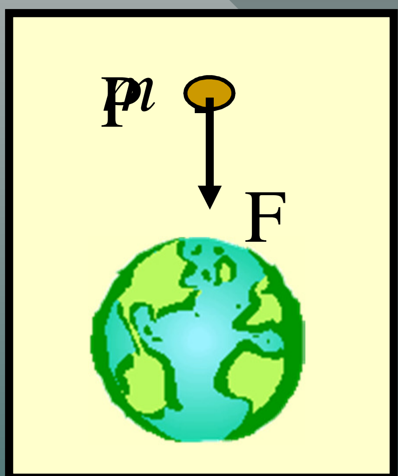
Electric Field

Objectives: After finishing this unit you should be able to:

- Define the electric field and explain what determines its magnitude and direction.
- Write and apply formulas for the electric field intensity at known distances from point charges.
- Discuss electric field lines and the meaning of permittivity of space.
- Write and apply Gauss's law for fields around surfaces of known charge densities.

The Concept of a Field

A field is defined as a property of space in which a material object experiences a force.



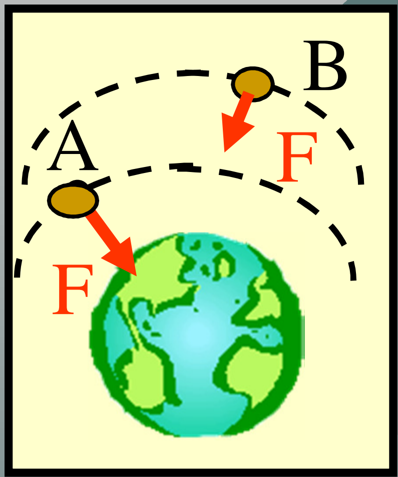
Above earth, we say there is a gravitational field at P.

Because a mass *m* experiences a downward force at that point.

No force, no field; No field, no force!

The direction of the field is determined by the force.

Consider points A and B above the surface of the earth—just points in space.



Note that the force F is real, but the field is just a convenient way of describing space.

The field at points A or B might be found from:

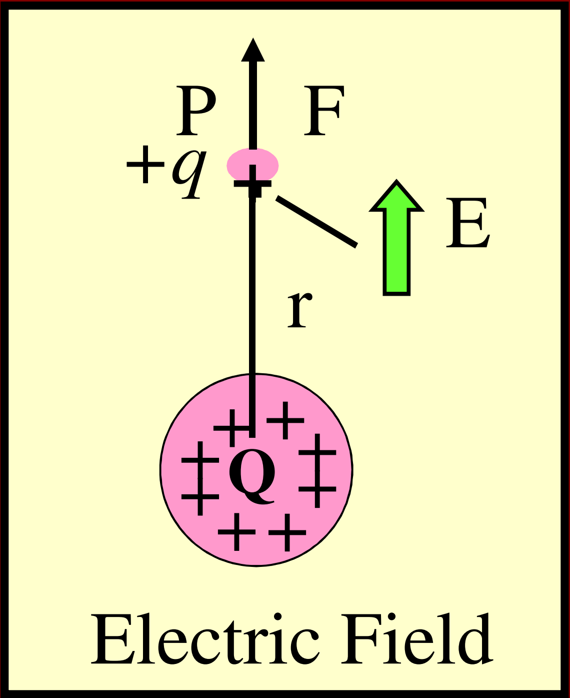
$$g = \frac{F}{m}$$

If g is known at every point above the earth then the force F on a given mass can be found.

The magnitude and direction of the field g is depends on the weight, which is the force F.

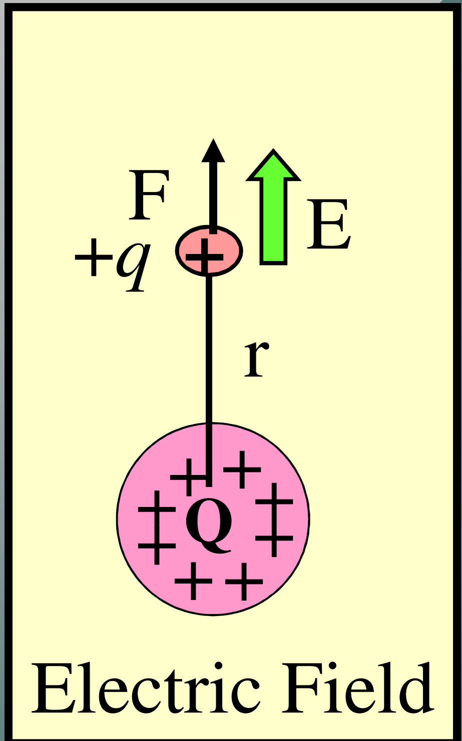
The Electric Field

1. Now, consider point P a distance *r* from +Q.
2. An electric field E exists at P if a test charge +q has a force F at that point.
3. The direction of the E is the same as the direction of a force on + (pos) charge.
4. The magnitude of E is given by the formula:



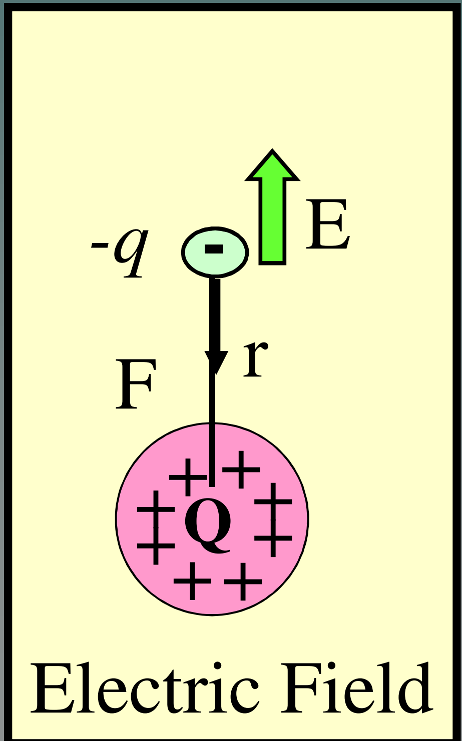
$$E = \frac{F}{q}; \text{ Units } \frac{\text{N}}{\text{C}}$$

Field is Property of Space



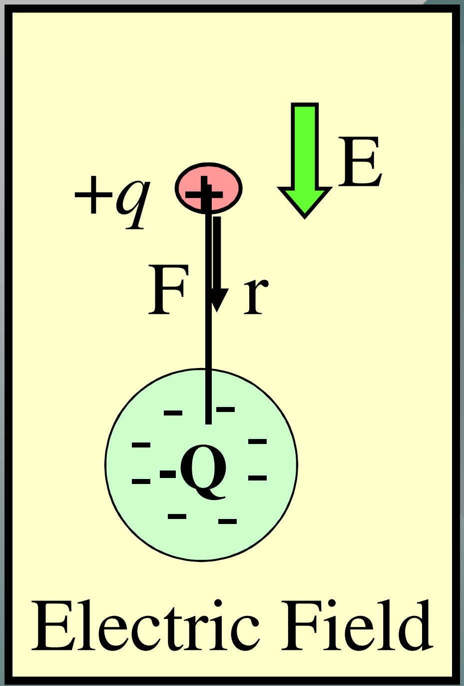
Force on +q is with field direction.

Force on -q is against field direction.



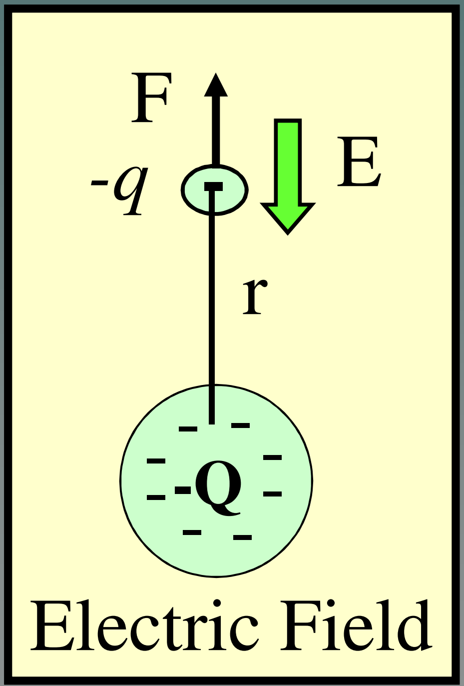
The field E at a point exists whether there is a charge at that point or not. The direction of the field is away from the +Q charge.

Field Near a Negative Charge



Force on +q is with field direction.

Force on -q is against field direction.



Note that the field E in the vicinity of a negative charge –Q is toward the charge—the direction that a +q test charge would move.

The Magnitude of E-Field

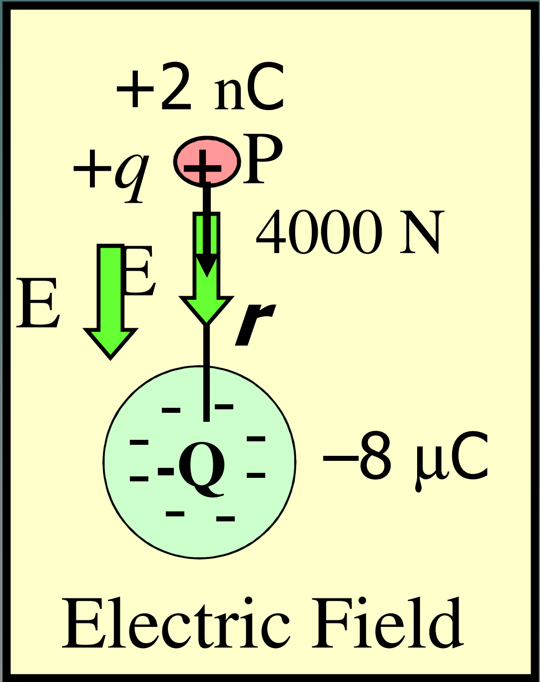
The magnitude of the electric field intensity at a point in space is defined as the force per unit charge (N/C) that would be experienced by any test charge placed at that point.

Electric Field Intensity E

$$E = \frac{F}{q}; \text{ Units } \left(\frac{\text{N}}{\text{C}}\right)$$

The direction of E at a point is the same as the direction that a positive charge would move IF placed at that point.

Example 1. A +2 nC charge is placed at a distance r from a –8 μC charge. If the charge experiences a force of 4000 N, what is the electric field intensity E at point P?



First, we note that the direction of E is toward –Q (down).

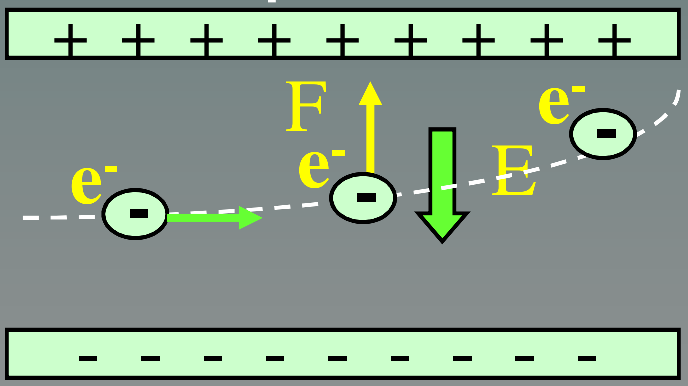
$$E = \frac{F}{q} = \frac{4000\text{ N}}{2 \times 10^{-9}\text{C}}$$

E = 2 x 10^{12} N/C Downward

Note: The field E would be the same for any charge placed at point P. It is a property of that space.

Example 2. A constant E field of 40,000 N/C is maintained between the two parallel plates. What are the magnitude and direction of the force on an electron that passes horizontally between the plates.c

The E-field is downward, and the force on e^- is up.



$$E = \frac{F}{q}; \quad F = qE$$

$$F = qE = (1.6 \text{ x } 10^{-19}\text{C})(4 \text{ x } 10^{4} \tfrac{N}{C})$$

F = 6.40 x 10^{-15} N, Upward

The E-Field at a distance r from a single charge Q

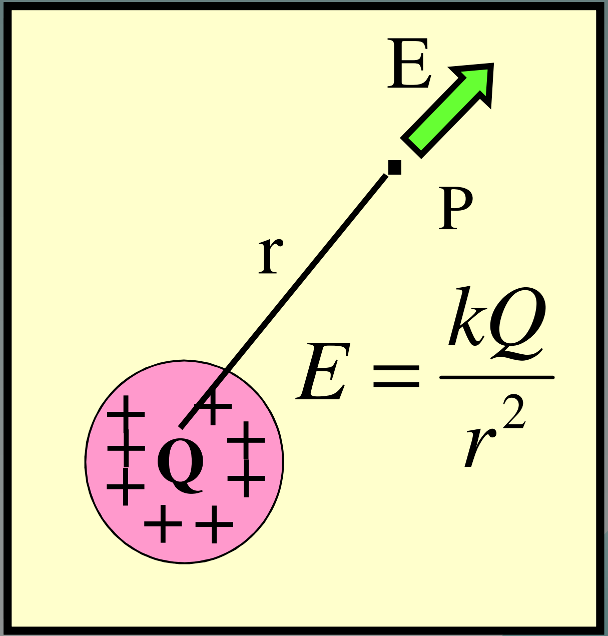
Consider a test charge +q placed at P a distance r from Q.

The outward force on +q is:

$$F = \frac{kQq}{r^2}$$

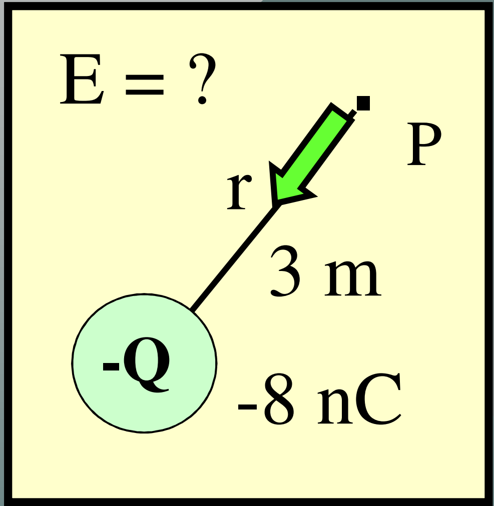
The electric field E is therefore:

$$E = \frac{F}{q} = \frac{kQq/r^2}{q}$$



$$E = \frac{kQ}{r^2}$$

Example 3. What is the electric field intensity E at point P, a distance of 3 m from a negative charge of –8 nC?



First, find the magnitude:

$$E = \frac{kQ}{r^2} = \frac{(9 \text{ x } 10^9 \frac{\text{Nm}^2}{\text{C}^2})(8 \text{ x } 10^{-9}\text{C})}{(3 \text{ m})^2}$$

E = 8.00 N/C

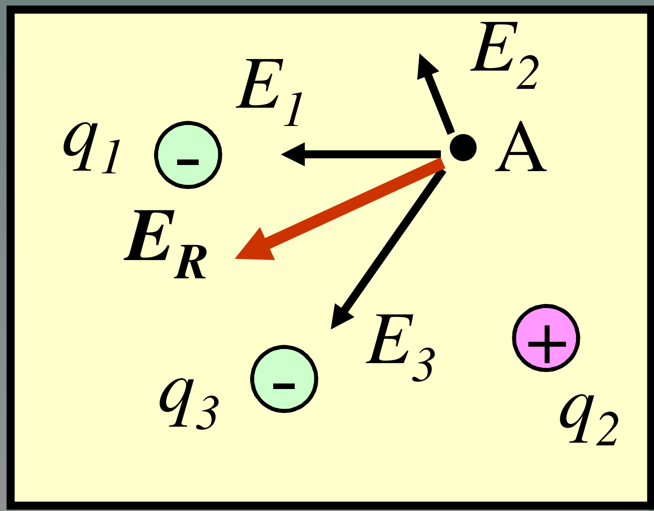
The direction is the same as the force on a positive charge if it were placed at the point P: toward –Q.

E = 8.00 N, toward -Q

The Resultant Electric Field.

The resultant field E in the vicinity of a number of point charges is equal to the vector sum of the fields due to each charge taken individually.

Consider E for each charge.



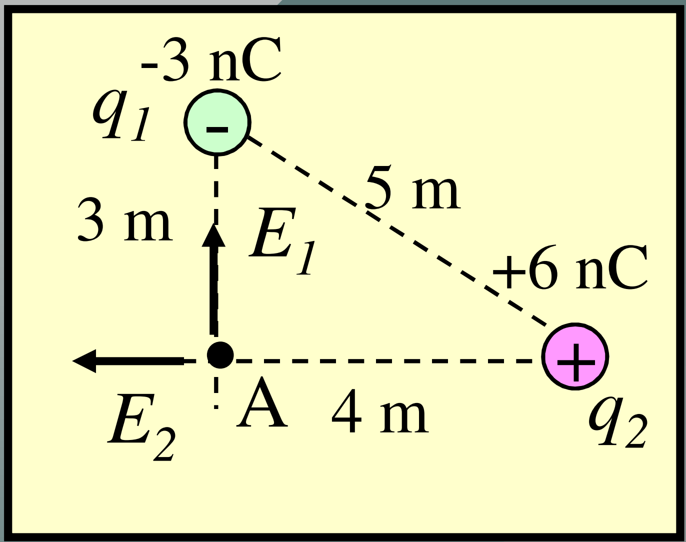
Vector Sum:
$\boldsymbol{E} = \boldsymbol{E}_1 + \boldsymbol{E}_2 + \boldsymbol{E}_3$

Magnitudes are from:

$$E = \frac{kQ}{r^2}$$

Directions are based on positive test charge.

Example 4. Find the resultant field at point A due to the –3 nC charge and the +6 nC charge arranged as shown.



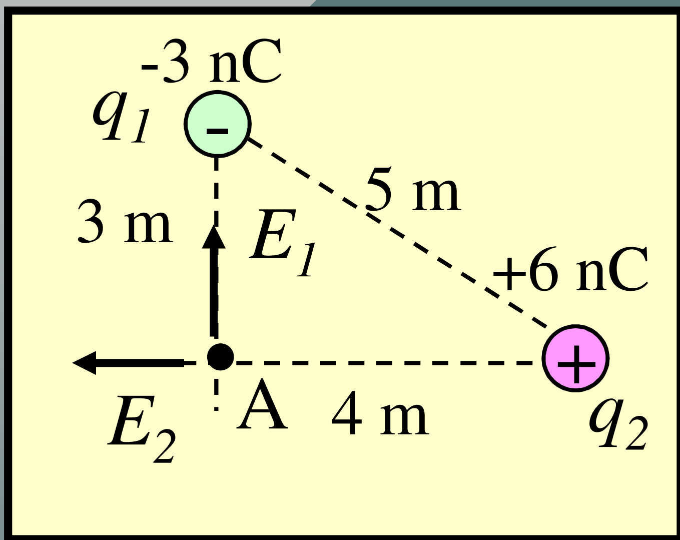
E for each q is shown with direction given.

$$E_1 = \frac{kq_1}{r_1^2}; \quad E_2 = \frac{kq_2}{r_2^2}$$

$$E_1 = \frac{(9 \text{ x } 10^9 \frac{\text{Nm}^2}{\text{C}^2})(3 \text{ x } 10^{-9}\text{C})}{(3 \text{ m})^2} \qquad E_2 = \frac{(9 \text{ x } 10^9 \frac{\text{Nm}^2}{\text{C}^2})(6 \text{ x } 10^{-9}\text{C})}{(4 \text{ m})^2}$$

Signs of the charges are used only to find <u>direction</u> of E

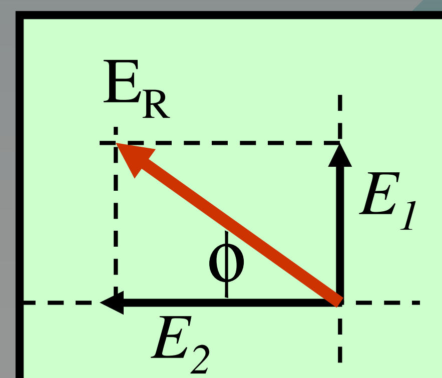
Example 4. (Cont.) Find the resultant field at point A. The magnitudes are:



$$E_1 = \frac{(9 \text{ x } 10^9 \frac{\text{Nm}^2}{\text{C}^2})(3 \text{ x } 10^{-9}\text{C})}{(3 \text{ m})^2}$$

$$E_2 = \frac{(9 \text{ x } 10^9 \frac{\text{Nm}^2}{\text{C}^2})(6 \text{ x } 10^{-9}\text{C})}{(4 \text{ m})^2}$$

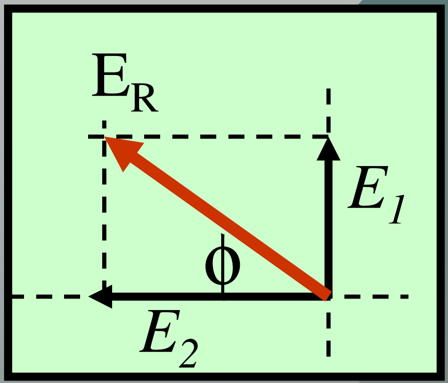
E_1 = 3.00 N, West $\quad$ E_2 = 3.38 N, North



Next, we find vector resultant E_R

$$E_R = \sqrt{E_2^2 + R_1^2}; \quad \tan\phi = \frac{E_1}{E_2}$$

Example 4. (Cont.) Find the resultant field at point A using vector mathematics.



E_1 = 3.00 N, West

E_2 = 3.38 N, North

Find vector resultant E_R

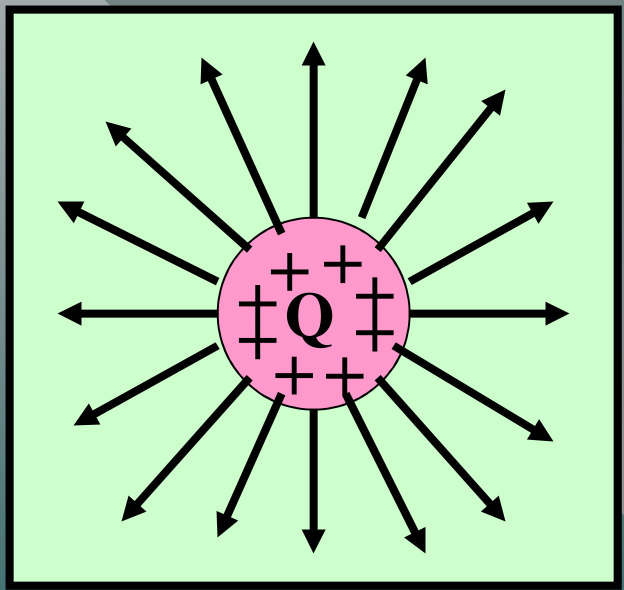
$$E = \sqrt{(3.00\text{ N})^2 + (3.38\text{ N})^2} = 4.52\text{ N}; \qquad \tan\phi = \frac{3.38\text{ N}}{3.00\text{ N}}$$

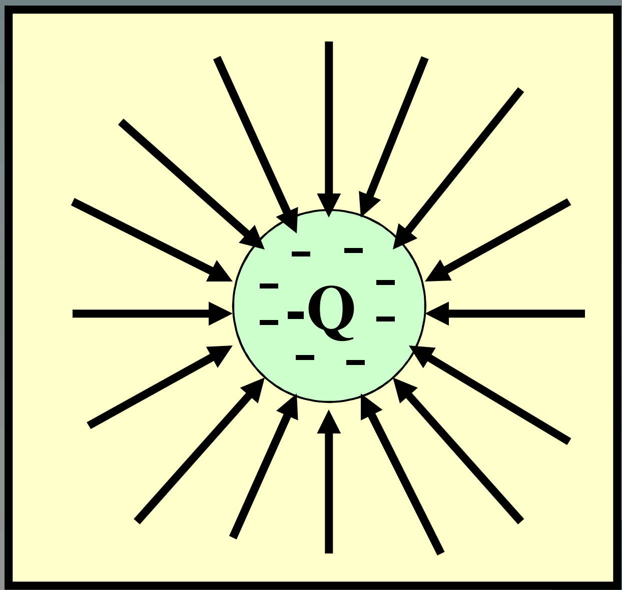
$\phi = 48.4^0$ N of W; or $\theta = 131.6^0$

Resultant Field: $E_R = 4.52$ N; 131.6^0

Electric Field Lines

Electric Field Lines are imaginary lines drawn in such a way that their direction at any point is the same as the direction of the field at that point.



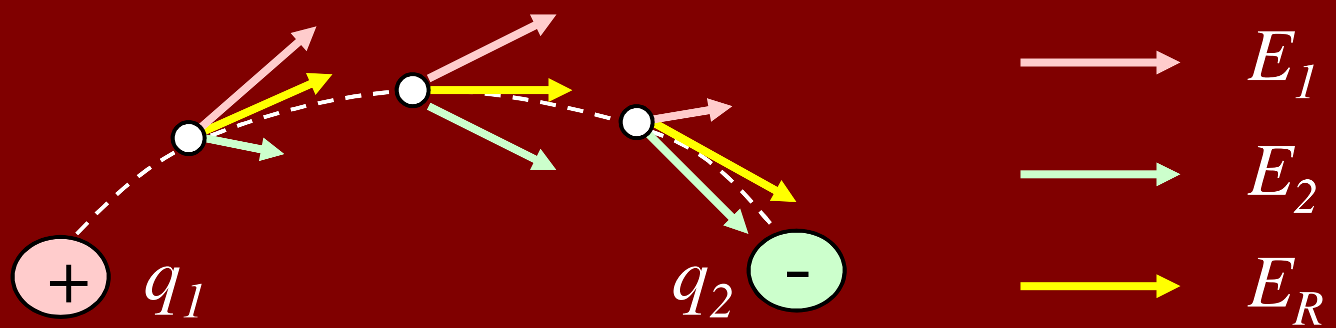


Field lines go away from positive charges and toward negative charges.

Rules for Drawing Field Lines

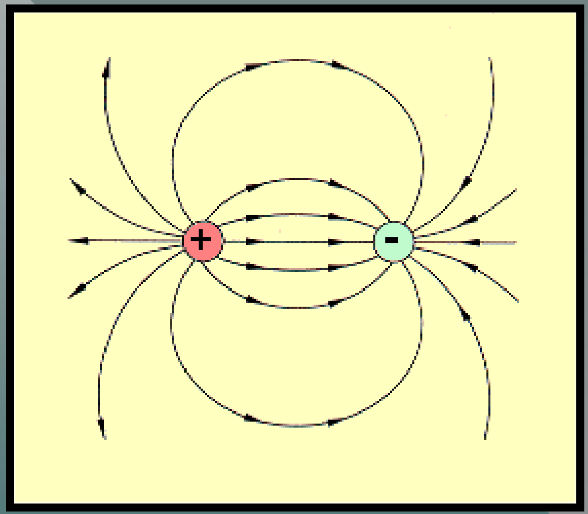
1. The direction of the field line at any point is the same as motion of +q at that point.

2. The spacing of the lines must be such that they are close together where the field is strong and far apart where the field is weak.

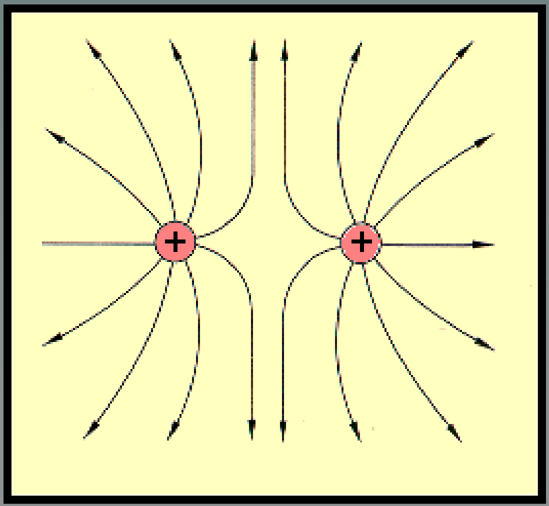


Examples of E-Field Lines

Two equal but opposite charges.



Two identical charges (both +).

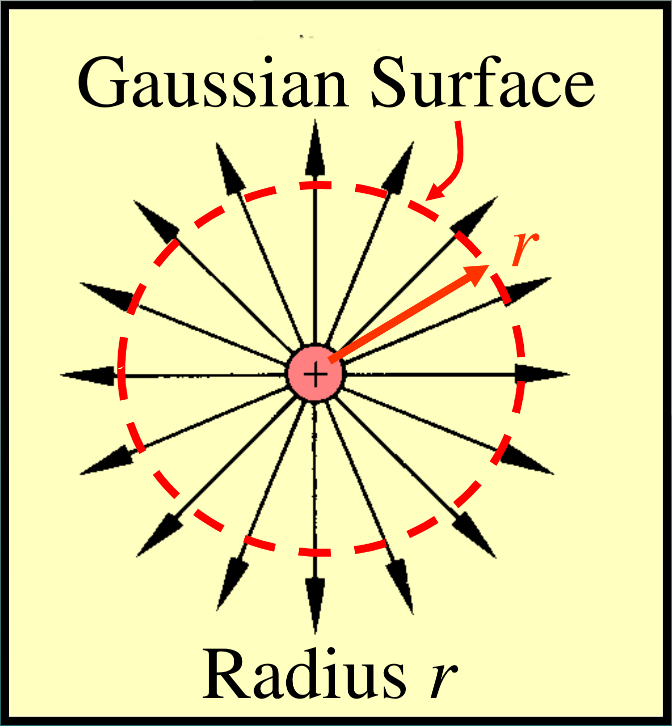


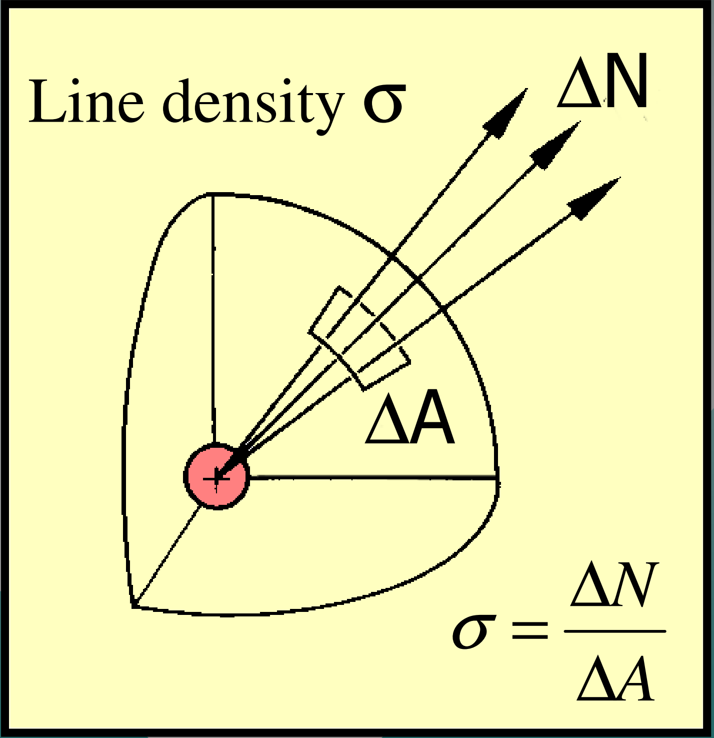
Notice that lines leave + charges and enter - charges.

Also, E is strongest where field lines are most dense.

The Density of Field Lines

Gauss's Law: The field E at any point in space is proportional to the line density σ at that point.

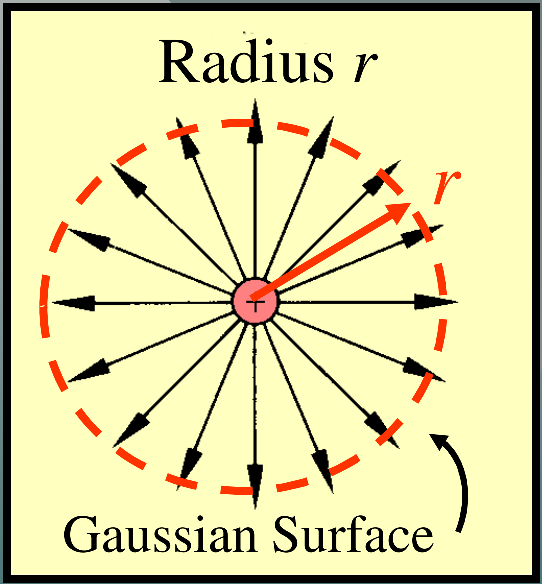




Line Density and Spacing Constant

Consider the field near a positive point charge q:

Then, imagine a surface (radius r) surrounding q.



E is proportional to $\Delta N/\Delta A$ and is equal to kq/r^2 at any point.

$$\frac{\Delta N}{\Delta A} \propto E; \qquad \frac{kq}{r^2} = E$$

Define ε_o as spacing constant. Then:

$$\frac{\Delta N}{\Delta A} = \varepsilon_0 E$$ Where ε_0 is:

$$\varepsilon_0 = \frac{1}{4\pi k}$$

Permittivity of Free Space

The proportionality constant for line density is known as the permittivity ε_o and it is defined by:

$$\varepsilon_0 = \frac{1}{4\pi k} = 8.85 \text{ x } 10^{-12} \ \frac{\text{C}^2}{\text{N} \cdot \text{m}^2}$$

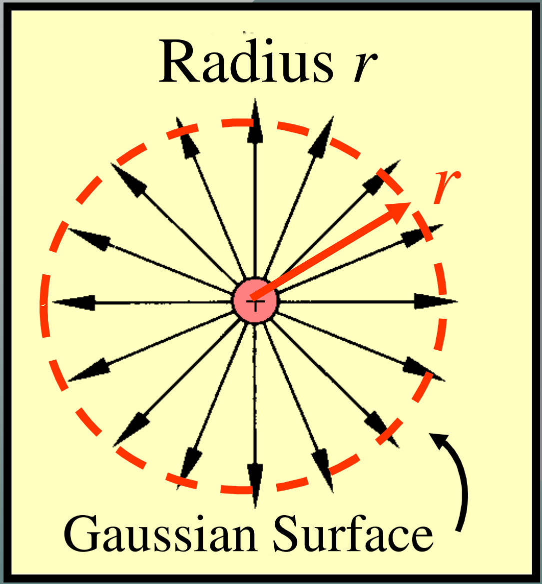
Recalling the relationship with line density, we have:

$$\frac{\Delta N}{\Delta A} = \varepsilon_0 E \quad or \quad \Delta N = \varepsilon_0 E \Delta A$$

Summing over entire area A gives the total lines as:

$$N = \varepsilon_o EA$$

Example 5. Write an equation for finding the total number of lines *N* leaving a single positive charge *q*.



Draw spherical Gaussian surface:

$$\Delta N = \varepsilon_0 E \Delta A \quad \text{and} \quad N = \varepsilon_0 \mathrm{EA}$$

Substitute for E and A from:

$$E = \frac{kq}{r^2} = \frac{q}{4\pi r^2}; \quad \mathrm{A} = 4\pi \mathrm{r}^2$$

$$N = \varepsilon_0 EA = \varepsilon_0 \left(\frac{q}{4\pi r^2} \right) (4\pi r^2)$$

$$N = \varepsilon_o qA = q$$

Total number of lines is equal to the enclosed charge q.

Gauss's Law

Gauss's Law: The net number of electric field lines crossing any closed surface in an outward direction is numerically equal to the net total charge within that surface.

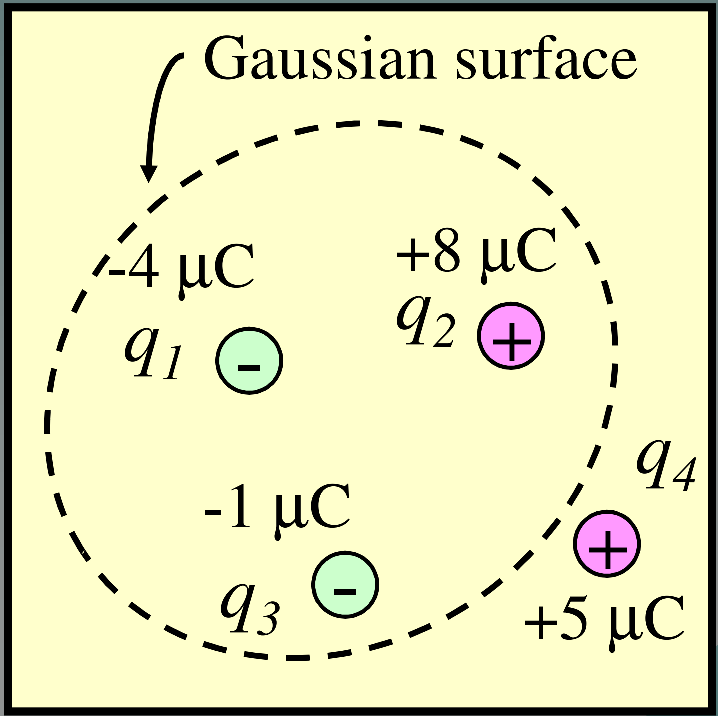
$$N = \Sigma \varepsilon_0 EA = \Sigma q$$

If we represent q as net enclosed positive charge, we can write rewrite Gauss's law as:

$$\Sigma EA = \frac{q}{\varepsilon_0}$$

Example 6. How many electric field lines pass through the Gaussian surface drawn below.

First we find the NET charge Σq enclosed by the surface:



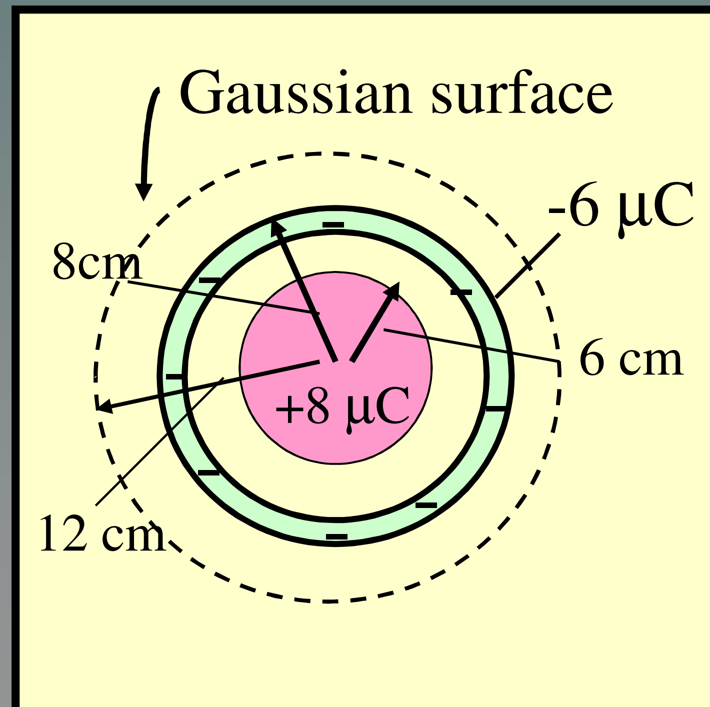
$$\Sigma q = (+8 - 4 - 1) = +3\ \mu C$$

$$N = \Sigma \varepsilon_0 EA = \Sigma q$$

N = +3 μC = +3 x 10^{-6} lines

Example 6. A solid sphere (R = 6 cm) having net charge +8 μC is inside a hollow shell (R = 8 cm) having a net charge of –6 μC. What is the electric field at a distance of 12 cm from the center of the solid sphere?

Draw Gaussian sphere at radius of 12 cm to find E.



$$N = \Sigma \varepsilon_0 EA = \Sigma q$$

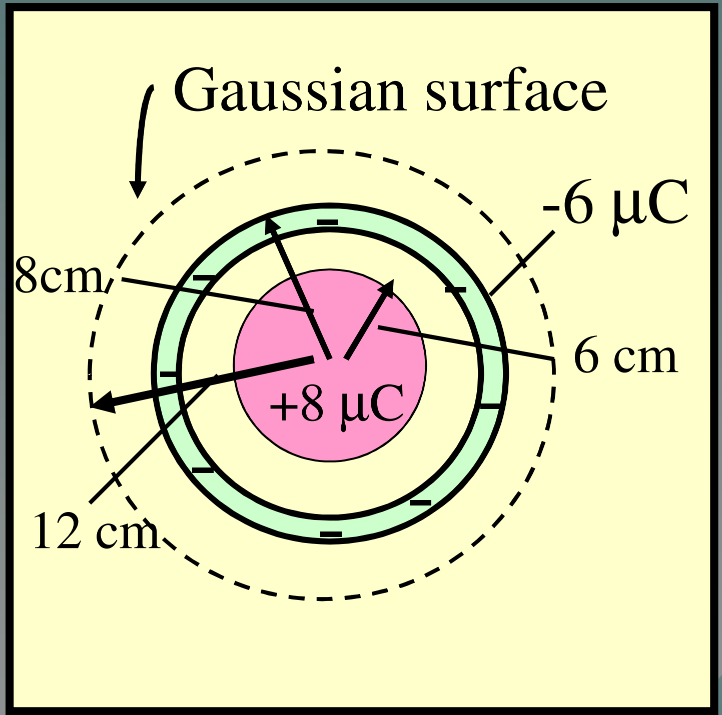
$$\Sigma q = (+8 - 6) = +2\ \mu C$$

$$\varepsilon_0 AE = q_{net};\ E = \frac{\Sigma q}{\varepsilon_0 A}$$

$$E = \frac{\Sigma q}{\varepsilon_0 (4\pi r^2)} = \frac{+2 \times 10^{-6} C}{(8.85 \times 10^{-12}\ \mathrm{Nm^2/C^2})(4\pi)(0.12\ m)^2}$$

Example 6 (Cont.) What is the electric field at a distance of 12 cm from the center of the solid sphere?

Draw Gaussian sphere at radius of 12 cm to find E.



$$N = \Sigma \varepsilon_0 EA = \Sigma q$$

$$\Sigma q = (+8 - 6) = +2\ \mu C$$

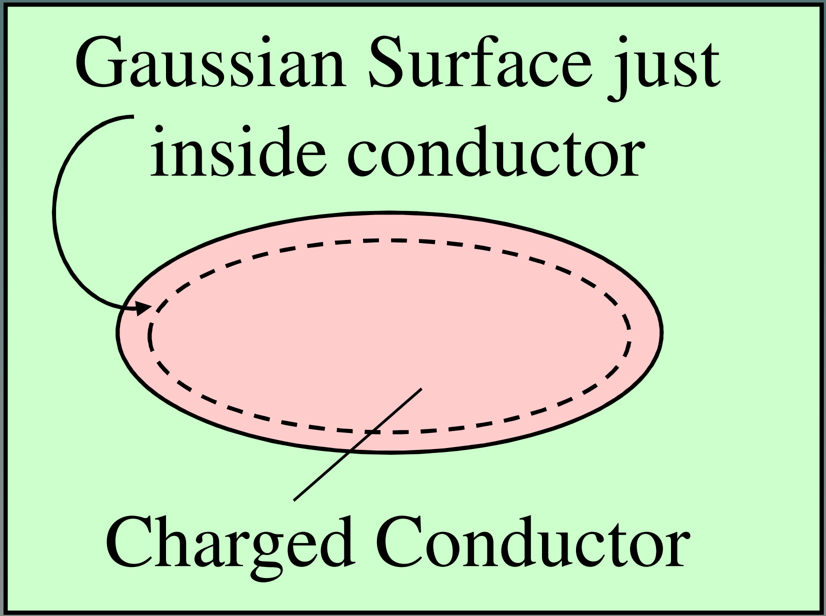
$$\varepsilon_0 AE = q_{net};\ E = \frac{\Sigma q}{\varepsilon_0 A}$$

$$E = \frac{+2\ \mu C}{\varepsilon_0 (4\pi r^2)} = 1.25 \times 10^6\ \text{N/C}$$

E = 1.25 MN/C

Charge on Surface of Conductor

Since like charges repel, you would expect that all charge would move until they come to rest. Then from Gauss's Law . . .



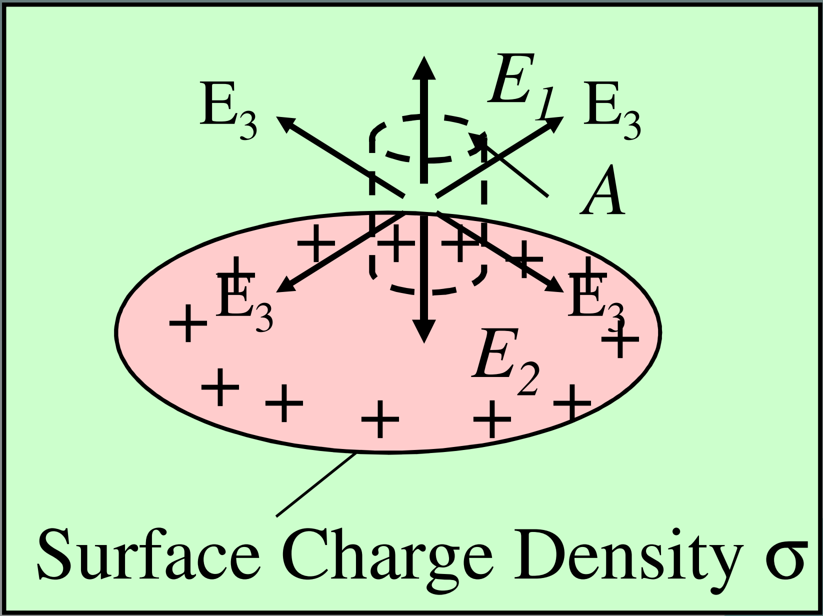
Since charges are at rest, E = 0 inside conductor, thus:

$$N = \Sigma \varepsilon_0 EA = \Sigma q \quad \text{or} \quad 0 = \Sigma q$$

All charge is on surface; None inside Conductor

Example 7. Use Gauss's law to find the E-field just outside the surface of a conductor. The surface charge density $\sigma = q/A$.

Consider q inside the pillbox. E-lines through all areas outward.



$$\Sigma \varepsilon_0 A E = q$$

E-lines through sides cancel by symmetry.

The field is zero inside the conductor, so $E_2 = 0$

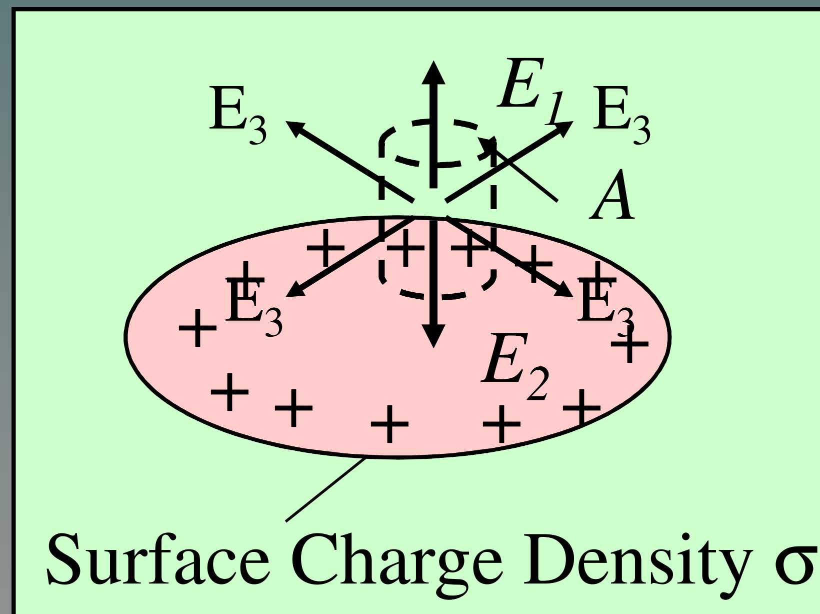
$$\varepsilon_0 E_1 A + \varepsilon_0 E_2 A = q \quad (\varepsilon_0 E_2 A \rightarrow 0)$$

$$E = \frac{q}{\varepsilon_0 A} = \frac{\sigma}{\varepsilon_0}$$

Example 7 (Cont.) Find the field just outside the surface if $\sigma = q/A = +2\ C/m^2$.

Recall that side fields cancel and inside field is zero, so that

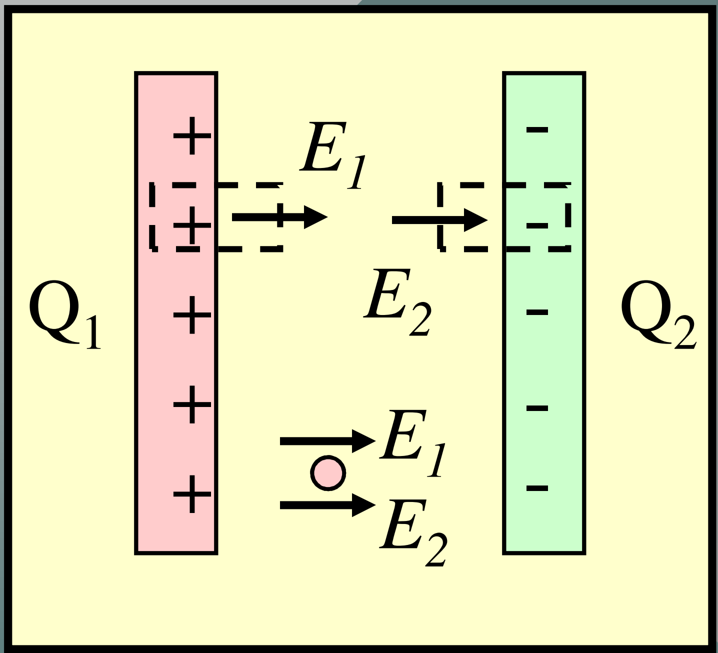
$$E_1 = \frac{q}{\varepsilon_0 A} = \frac{\sigma}{\varepsilon_0}$$



$$E = \frac{+2 \text{ x } 10^{-6} \text{C/m}^2}{8.85 \text{ x } 10^{-12}\ \text{Nm}^2/\text{C}^2}$$

E = 226,000 N/C

Field Between Parallel Plates



Equal and opposite charges.

Field E_1 and E_2 to right.

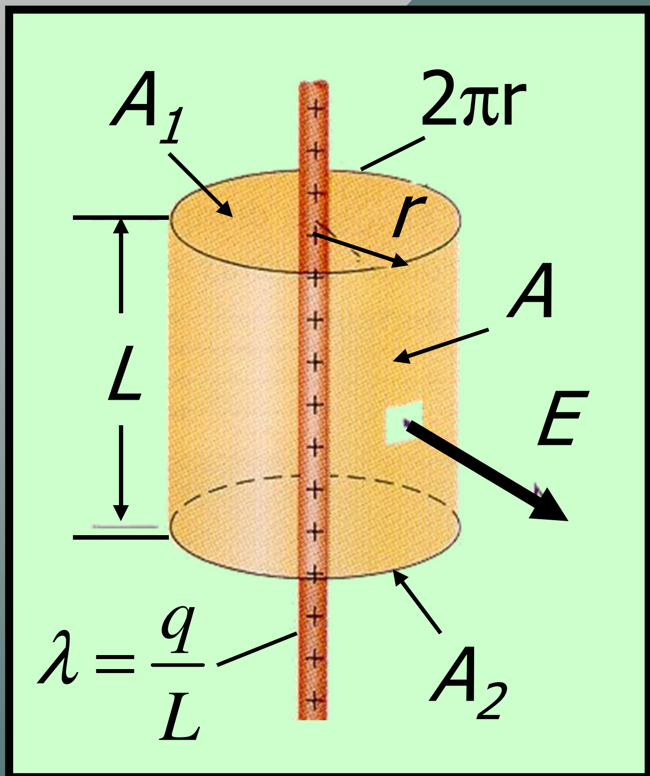
Draw Gaussian pillboxes on each inside surface.

Gauss's Law for either box gives same field ($E_1 = E_2$).

$$\Sigma \varepsilon_0 A E = \Sigma q$$

$$E = \frac{q}{\varepsilon_0 A} = \frac{\sigma}{\varepsilon_0}$$

Line of Charge



Field due to A_1 and A_2 Cancel out due to symmetry.

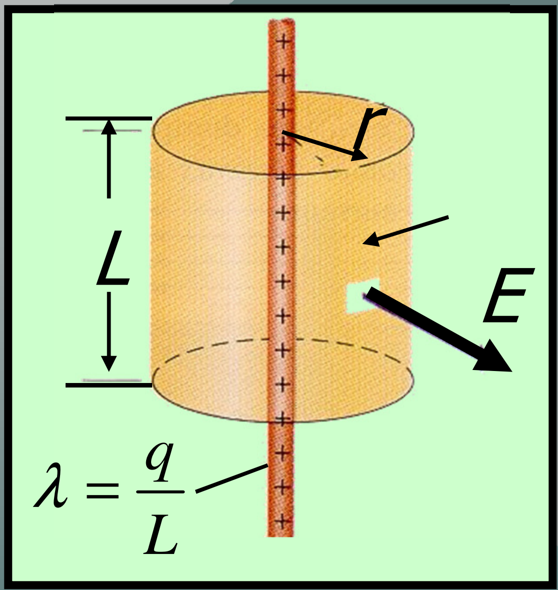
$$\Sigma \varepsilon_0 A E = q$$

$$EA = \frac{q}{\varepsilon_0};\ A = (2\pi r)L$$

$$E = \frac{q}{2\pi\varepsilon_0 r L};\quad \lambda = \frac{\mathrm{q}}{\mathrm{L}}$$

$$E = \frac{\lambda}{2\pi\varepsilon_0 r}$$

Example 8: The Electric field at a distance of 1.5 m from a line of charge is 5 x 10^4 N/C. What is the linear density of the line?



$$E = \frac{\lambda}{2\pi\varepsilon_0 r}$$

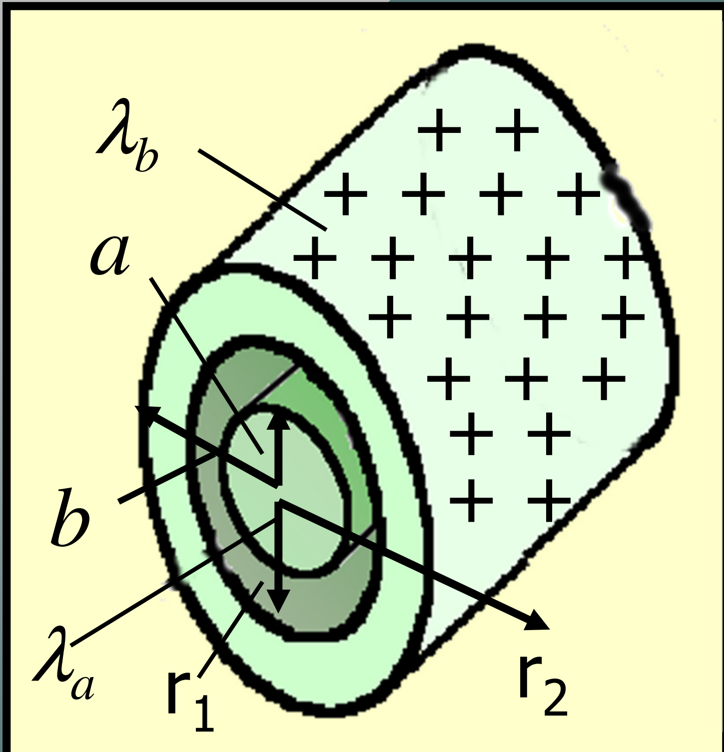
$$\lambda = 2\pi\varepsilon_0 rE$$

E = 5 x 10^4 N/C   r = 1.5 m

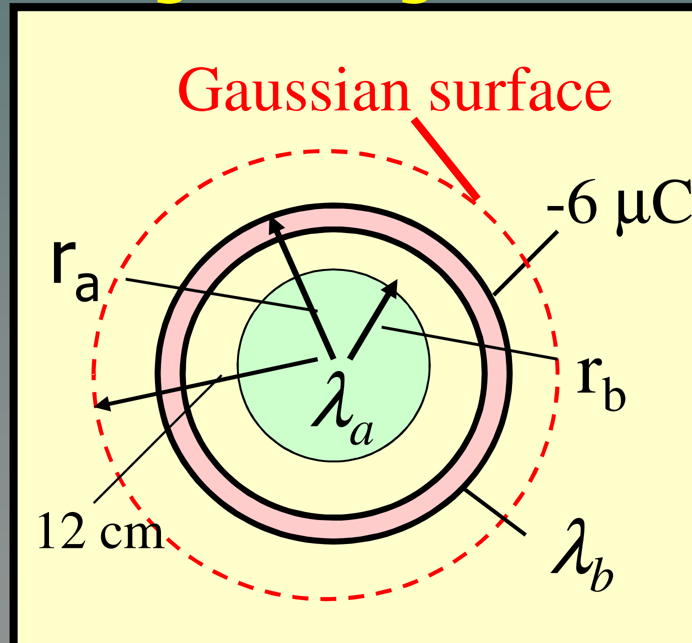
$$\lambda = 2\pi(8.85 \text{ x } 10^{-12} \tfrac{\text{C}^2}{\text{Nm}^2})(1.5 \text{ m})(5 \text{ x } 10^4 \text{N/C})$$

$$\lambda = 4.17\ \mu\text{C/m}$$

Concentric Cylinders



Outside is like charged long wire:



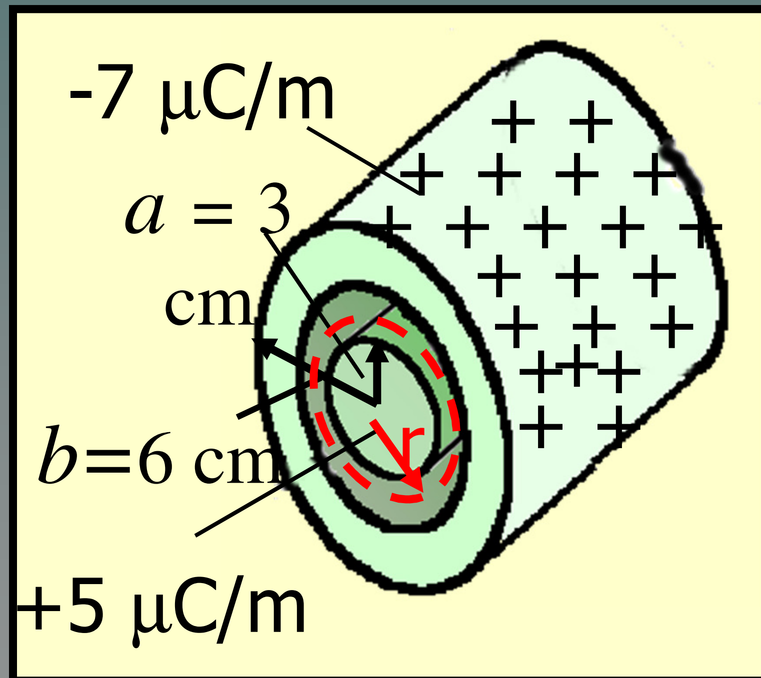
For $r > r_b$

$$E = \frac{\lambda_a + \lambda_b}{2\pi\varepsilon_0 r}$$

For $r_b > r > r_a$

$$E = \frac{\lambda_a}{2\pi\varepsilon_0 r}$$

Example 9. Two concentric cylinders of radii 3 and 6 cm. Inner linear charge density is +3 μC/m and outer is -5 μC/m. Find E at distance of 4 cm from center.



Draw Gaussian surface between cylinders.

$$E = \frac{\lambda_b}{2\pi\varepsilon_0 r}$$

$$E = \frac{+3\mu\text{C/m}}{2\pi\varepsilon_0 (0.04\text{ m})}$$

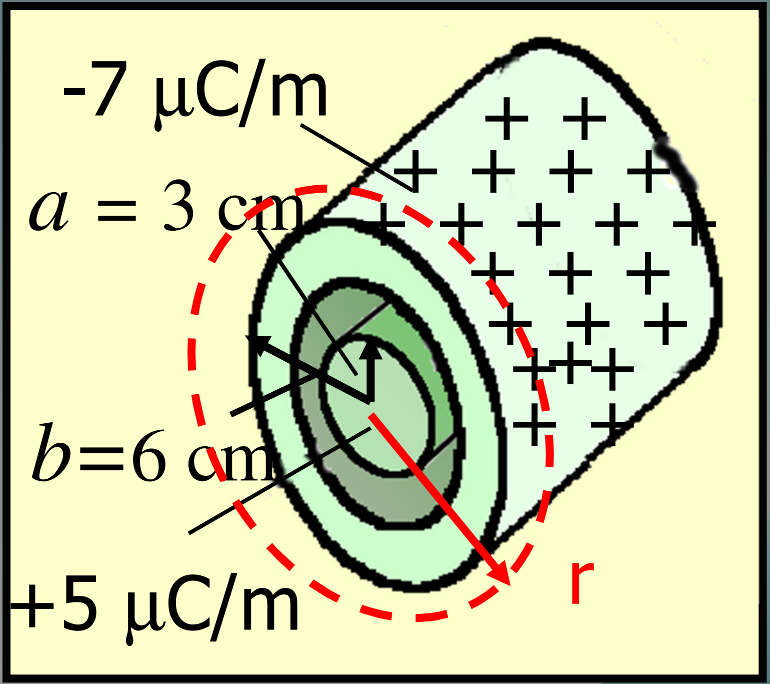
E = 1.38 x 10^6 N/C, Radially out

Example 8 (Cont.) Next, find E at a distance of 7.5 cm from center (outside both cylinders.)

Gaussian outside of both cylinders.

$$E = \frac{\lambda_a + \lambda_b}{2\pi\varepsilon_0 r}$$

$$E = \frac{(+3-5)\mu\text{C/m}}{2\pi\varepsilon_0 (0.075\text{ m})}$$



E = 5.00 x 10^5 N/C, Radially inward

Summary of Formulas

The Electric Field Intensity *E*:

$$E = \frac{F}{q} = \frac{kQ}{r^2} \qquad \text{Units are } \frac{\text{N}}{\text{C}}$$

The Electric Field Near several charges:

$$E = \sum \frac{kQ}{r^2} \qquad \text{Vector Sum}$$

Gauss's Law for Charge distributions.

$$\Sigma \varepsilon_0 EA = \Sigma q; \quad \sigma = \frac{q}{A}$$

CONCLUSION: Chapter 16 The Electric Field