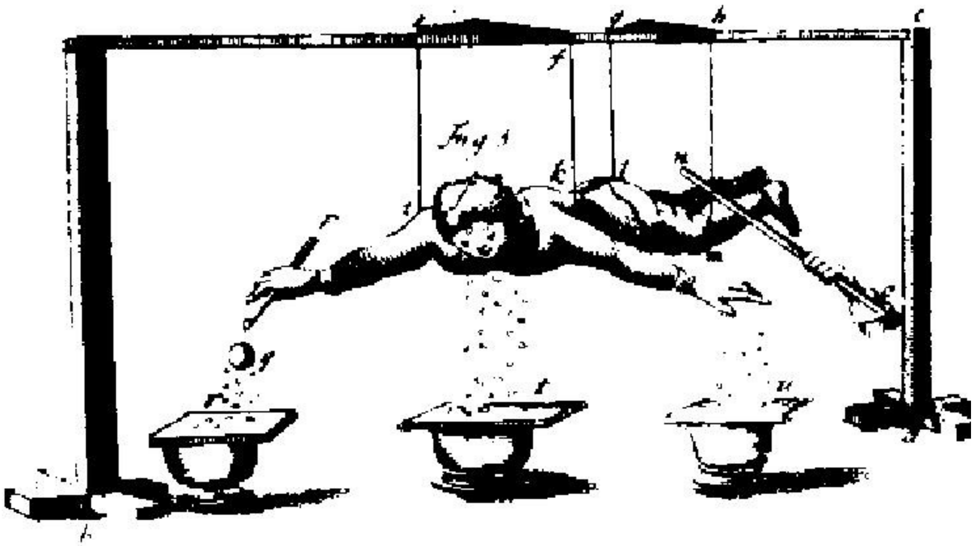
Electric Charge and Electric Field

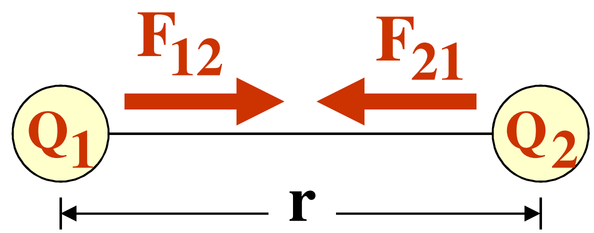


AP Physics 4
Lecture Notes

- **Coulomb's Law**
- **The Electric Field**
- **Field Lines**
- **Electric Fields and Conductors**

Coulomb's Law

Coulomb's law:



$$F = |F_{12}| = |F_{21}|$$

Force (N)

$$F = k\frac{Q_1 Q_2}{r^2}$$

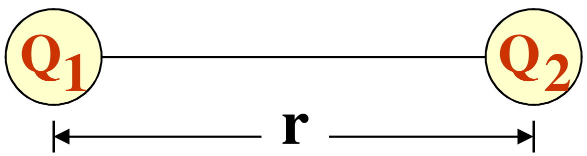
Charge (Q)
Coulombs (C)
1 C = 6.2421 x 10^{18} e
e = 1.602 x 10^{-19} C

Distance (m)

Coulomb's constant (k)
k = 8.988 x 10^{9} N·m^2/C^2

Coulomb's Law

Coulomb's law:



$$F = k\frac{Q_1 Q_2}{r^2}$$

$$k = \frac{1}{4\pi\varepsilon_o}$$

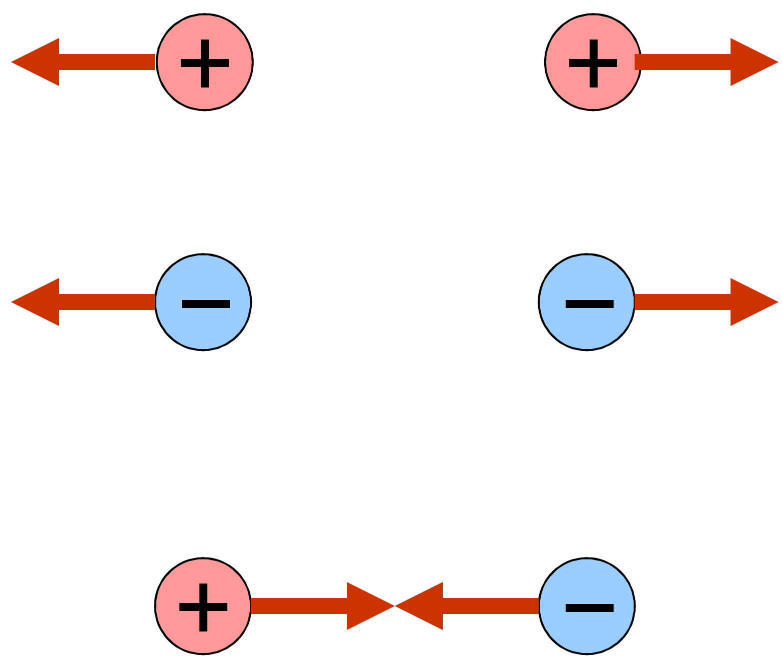
$$F = \frac{1}{4\pi\varepsilon_o}\frac{Q_1 Q_2}{r^2}$$

$$\varepsilon_o = \frac{1}{4\pi k} = 8.85 \times 10^{-12}\ \frac{C^2}{N \cdot m^2}$$

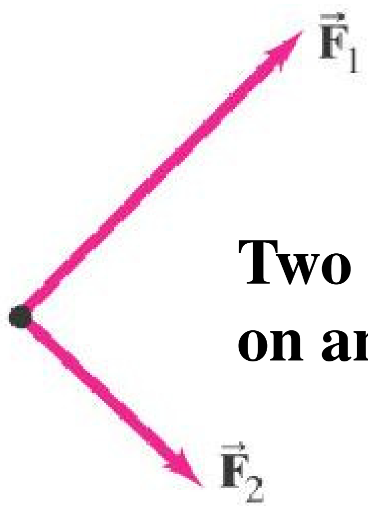
Permittivity of free space

Coulomb's Law

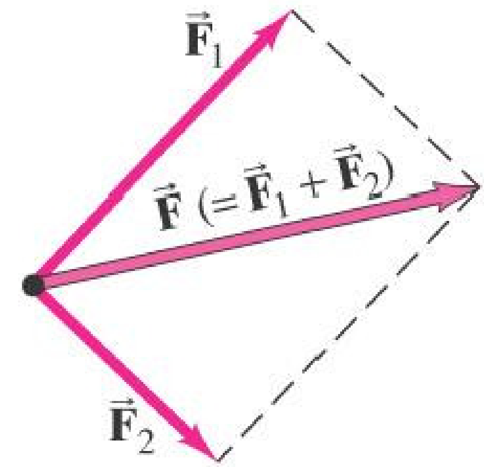
The force is along the line connecting the charges, and is attractive if the charges are opposite, and repulsive if they are the same.



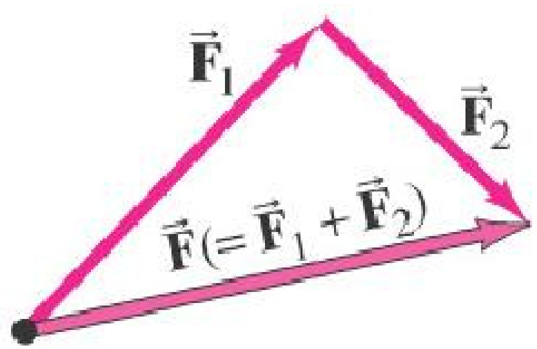
Vector addition review:



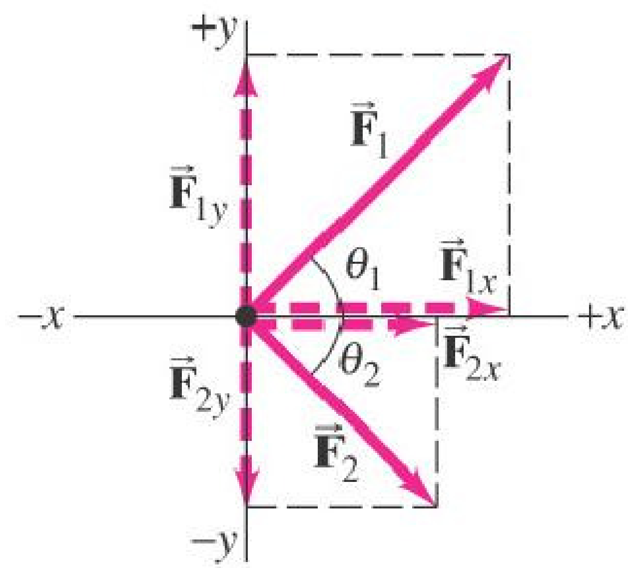
Two forces acting on an object



Parallelogram method



tail-to-tip method

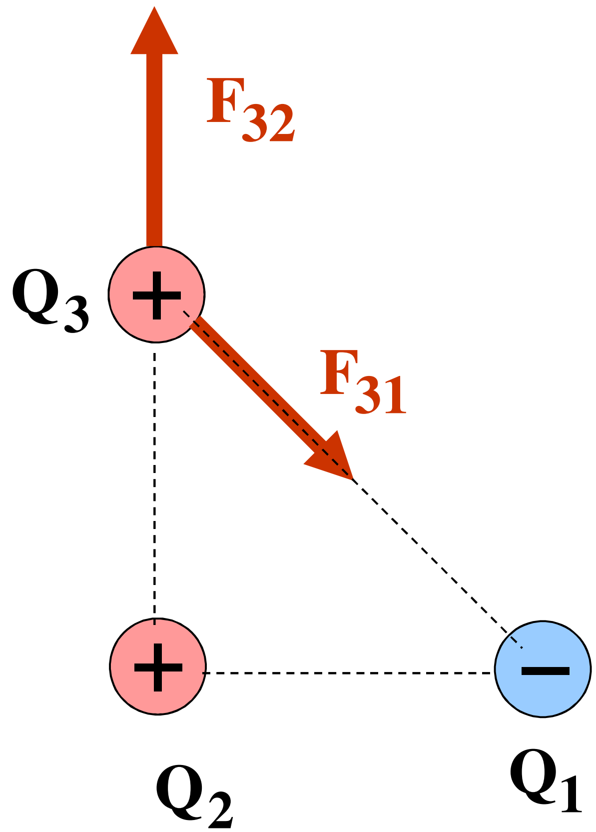


Components method

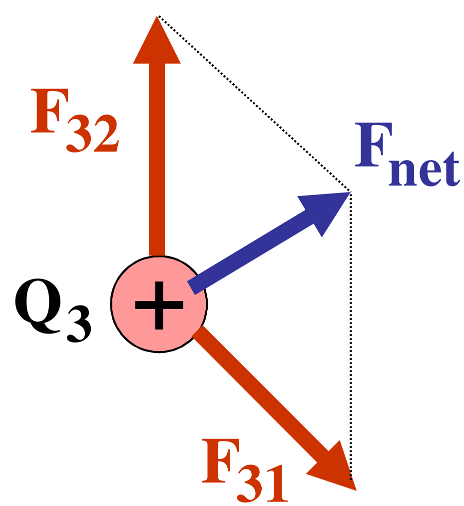
Coulomb's Law

Coulomb's law strictly applies only to point charges.

Superposition: for multiple point charges, the forces on each charge from every other charge can be calculated and then added as vectors.



Net force on Q_3



Problem 16-06

Two charged dust particles exert a force of .32 N on each other. What will be the force if they are moved so they are only one-eighth as far apart?

Coulomb's law is an inverse square law.

$$F = 64\left(.32\ N\right) = 20.48\ \mathrm{N}$$

Problem 16-07

A person scuffing her feet on a wool rug on a dry day accumulates a net charge of –42 μC. How many excess electrons does she get,

$$-42 \times 10^{-6}\ \mathrm{C}\left(\frac{1\ \text{electron}}{-1.602 \times 10^{-19}\ \mathrm{C}}\right) = \boxed{2.622 \times 10^{14}\ \text{electrons}}$$

By how much does her mass increase?

$$2.622 \times 10^{14}\ e^{-}\left(\frac{9.11 \times 10^{-31}\ \mathrm{kg}}{1\ e^{-}}\right) = \boxed{2.389 \times 10^{-16}\ \mathrm{kg}}$$

Problem 16-08

Compare the electric force holding the electron in orbit (r = 0.53 × 10 l0 m) around the proton nucleus of the hydrogen atom, with the gravitational force between the same electron and proton.
What is the ratio of these two forces?

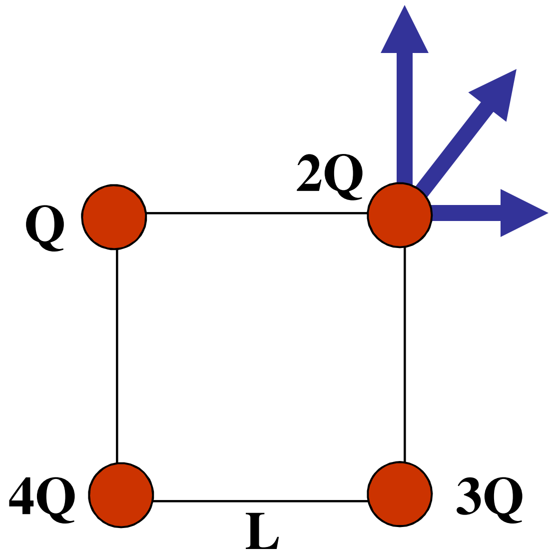
$$\frac{F_E}{F_G} = \frac{\frac{kQ_1Q_2}{r^2}}{\frac{Gm_1m_2}{r^2}} = \frac{kQ_1Q_2}{Gm_1m_2}$$

$$= \frac{8.988 \text{ x } 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2 \left(1.602 \text{ x } 10^{-19} \text{ C}\right)^2}{6.67 \text{ x } 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2 \left(9.11 \text{ x } 10^{-31} \text{kg}\right)\left(1.67 \text{ x } 10^{-27} \text{kg}\right)}$$

$$\boxed{= 2.3 \text{ x } 10^{39}}$$

Problem 16-16

At each corner of a square of side L there are four point charges. Determine the force on the charge 2Q.

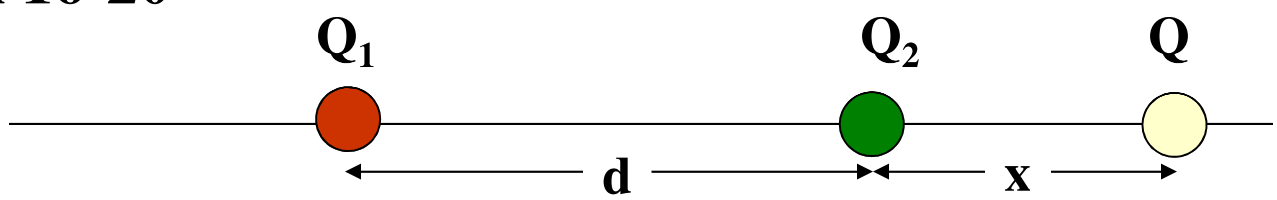


$$F_x = \frac{k(2Q)Q}{L^2} + \frac{k(2Q)4Q}{2L^2}\cos 45^o = \frac{kQ^2}{L^2}\left(2+2\sqrt{2}\right) = 4.8284\frac{kQ^2}{L^2}$$

$$F_y = \frac{k(2Q)3Q}{L^2} + \frac{k(2Q)4Q}{2L^2}\sin 45^o = \frac{kQ^2}{L^2}\left(6+2\sqrt{2}\right) = 8.8284\frac{kQ^2}{L^2}$$

$$F_{2Q} = \sqrt{F_x^2 + F_y^2} = \boxed{10.0625\frac{kQ^2}{L^2}}$$

Problem 16-20



A +4.75 μC and a –3.55 μC charge are placed 18.5 cm apart. Where can a third charge be placed so that it experiences no net force?

$$|F_1| = |F_2|$$

$$k\frac{Q_1 Q}{(d+x)^2} = k\frac{Q_2 Q}{x^2}$$

$$x = d\frac{\sqrt{Q_2}}{\sqrt{Q_1} - \sqrt{Q_2}}$$

$$x = 18.5\text{ cm}\frac{\sqrt{3.5 \times 10^{-6}\text{ C}}}{\sqrt{4.7 \times 10^{-6}\text{ C}} - \sqrt{3.5 \times 10^{-6}\text{ C}}}$$

$$= 116\text{ cm}$$

The Electric Field

The electric field is defined as the force on a test charge (small positive charge), divided by the charge:

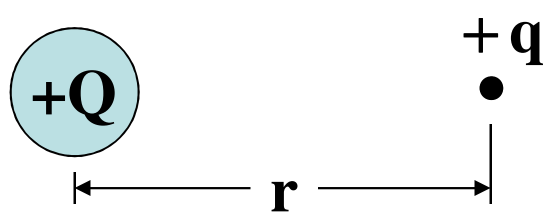
$$E = \frac{F}{q}$$

$$F = qE$$

$$F = -qE$$

The Electric Field

Electric field of a point charge



$$F = k\frac{Qq}{r^2}$$

$$E = \frac{F}{q} = \frac{k\frac{Qq}{r^2}}{q}$$

$$E = k\frac{Q}{r^2}$$

The Electric Field

Problem solving in electrostatics:

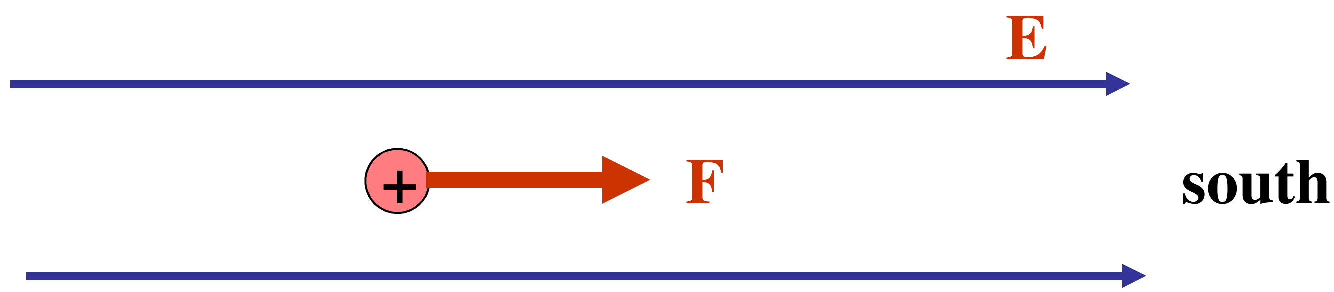
Draw a diagram; show all charges, with signs, and electric fields and forces with directions

Calculate forces using Coulomb's law

Add forces vectorially to get result

Problem 16-24

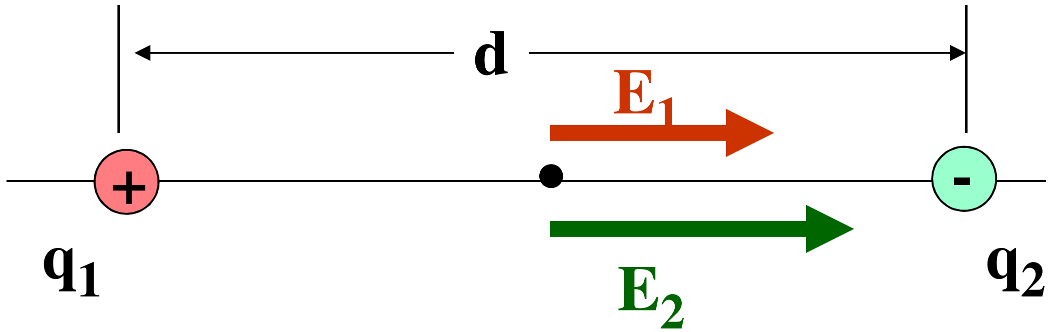
A proton is released in a uniform electric field, and it experiences an electric force of 3.75×10^{-14} N toward the south. What are the magnitude and direction of the electric field?



$$E = \frac{F}{q} = \frac{3.75 \times 10^{-14}\ \text{N}}{1.602 \times 10^{-19}\ \text{C}} = \boxed{2.34 \times 10^{5}\ \text{N/C}}$$

Problem 16-28

What are the magnitude and direction of the electric field at a point midway between a +7.0 μC and a –8.0 μC charge 8.0 cm apart? Assume no other charges are nearby.



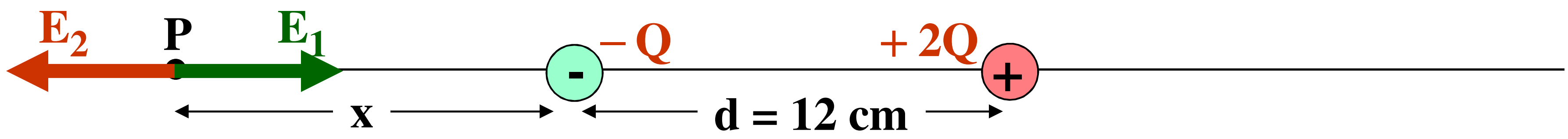
$$E = E_1 + E_2$$

$$E = \frac{kq_1}{\left(d/2\right)^2} + \frac{kq_2}{\left(d/2\right)^2}$$

$$E = \frac{4k}{d^2}\left(q_1 + q_2\right) = \frac{4\left(8.988\,\text{x}10^9\ \text{N}\cdot\text{m/C}\right)}{\left(0.08\,\text{m}\right)^2}\left(7.0\,\text{x}10^{-6}\ \text{N} + 8.0\,\text{x}10^{-6}\ \text{N}\right)$$

$$\boxed{E = 8.4\,\text{x}10^7\ \text{N/C}}$$

Problem 16-36



Two point charges, –Q and +2Q, are separated by a distance of d = 12 cm. The electric field at the point is zero. How far from –Q is P?

$$|E_1| = |E_2|$$

$$\frac{kQ_1}{x^2} = \frac{kQ_2}{(x+d)^2}$$

$$x = d\frac{\sqrt{Q_1}}{\sqrt{Q_2} - \sqrt{Q_1}}$$

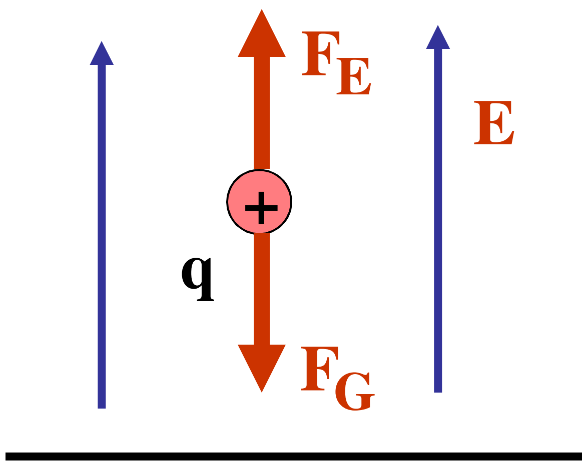
$$x = d\frac{\sqrt{Q_1}}{\sqrt{2Q_1} - \sqrt{Q_1}}$$

$$x = \frac{d}{\sqrt{2}-1} = \frac{12\,\text{cm}}{\sqrt{2}-1}$$

$$\boxed{x = 29\,\text{cm}}$$

Problem 16-54

A proton (m = 1.67 x 10^{-27} kg) is suspended at rest in a uniform electric field E. Take into account gravity at the Earth's surface, and determine E.



$$F_E = F_G$$

$$qE = mg$$

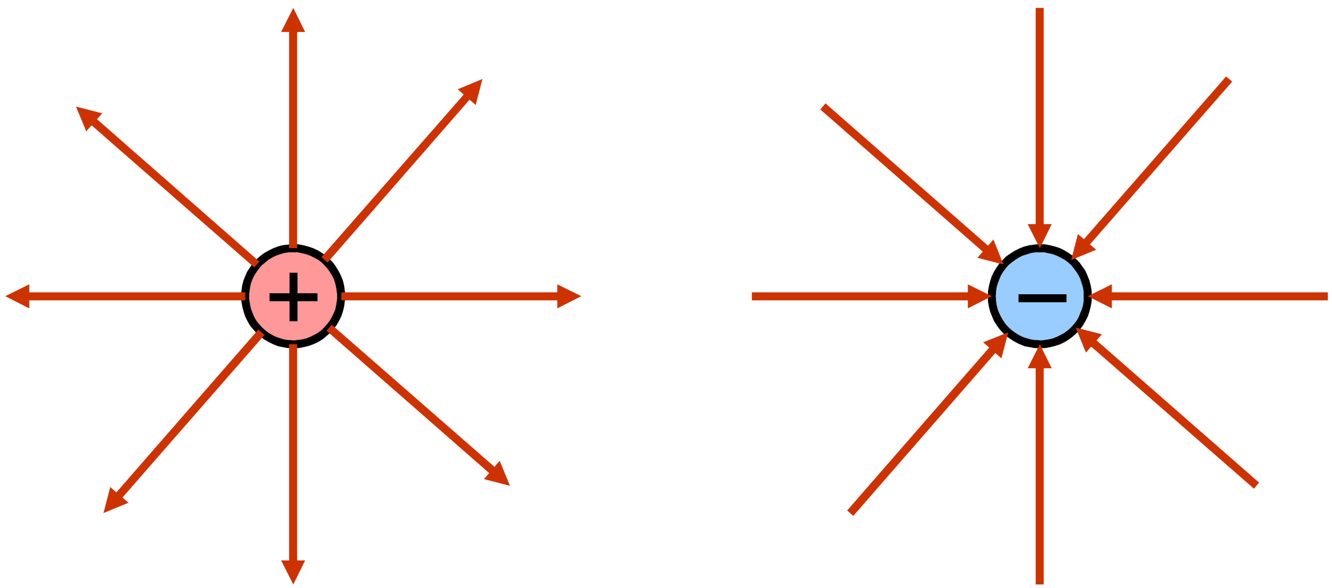
$$E = \frac{mg}{q}$$

$$E = \frac{1.67 \times 10^{-27}\ \text{kg}\left(9.80\ \text{m/s}^2\right)}{1.602 \times 10^{-19}\ \text{C}}$$

$$\boxed{E = 1.02 \times 10^{-7}\ \text{N/C}}$$

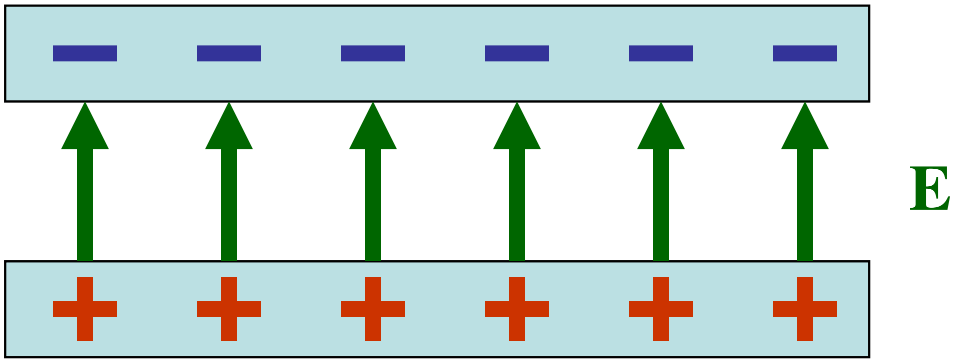
The Electric Field

The electric field can be represented by field lines. These lines start on a positive charge and end on a negative charge.

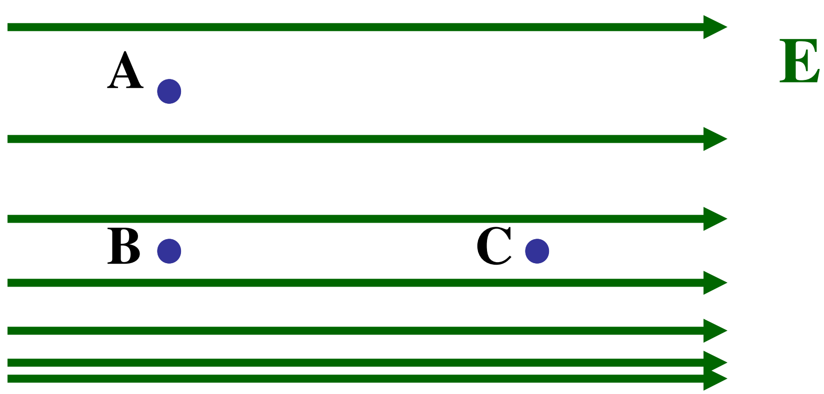


The Electric Field

The number of field lines starting (ending) on a positive (negative) charge is proportional to the magnitude of the charge.

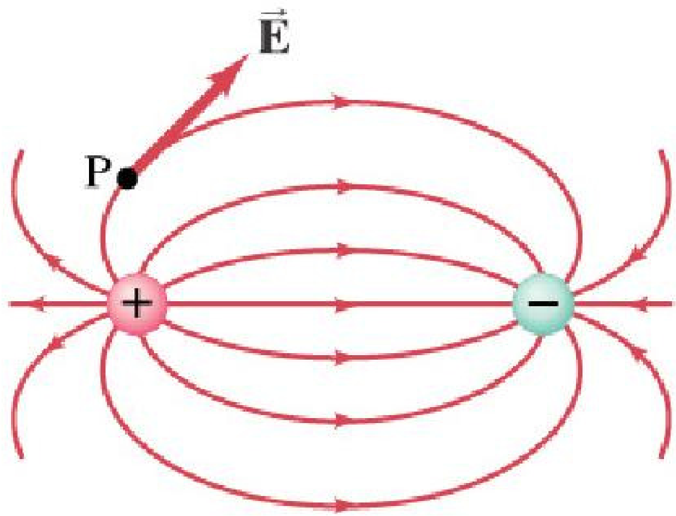


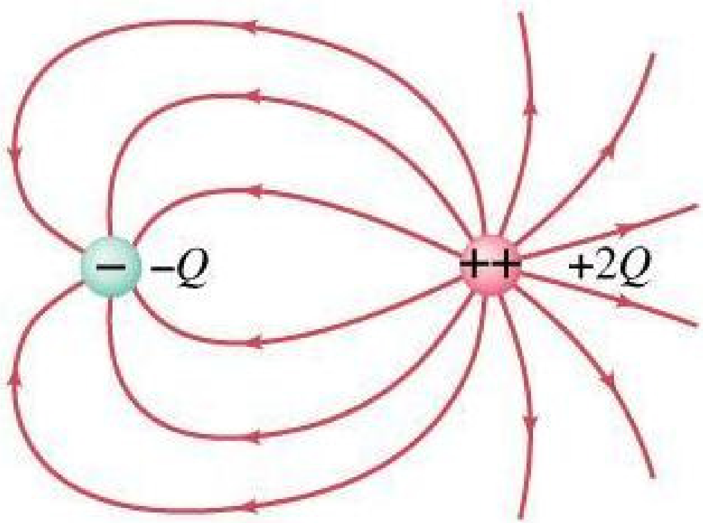
The electric field is stronger where the field lines are closer together.

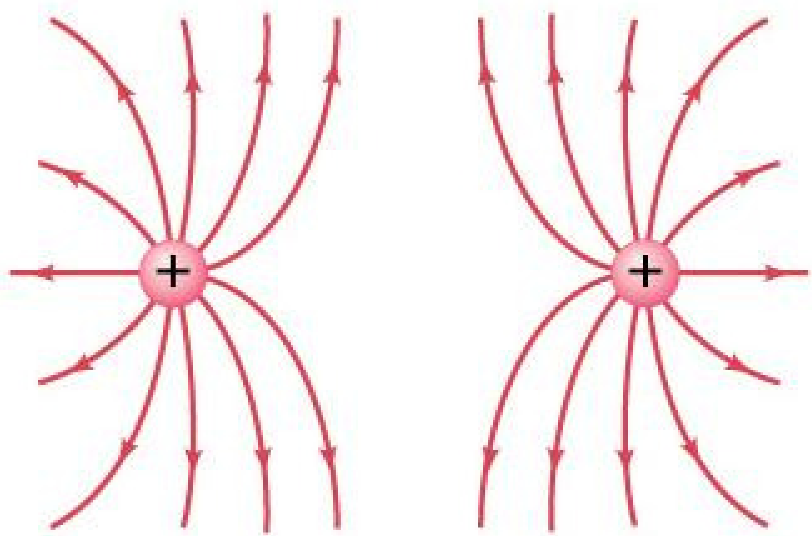


The Electric Field

Electric field of two charges:

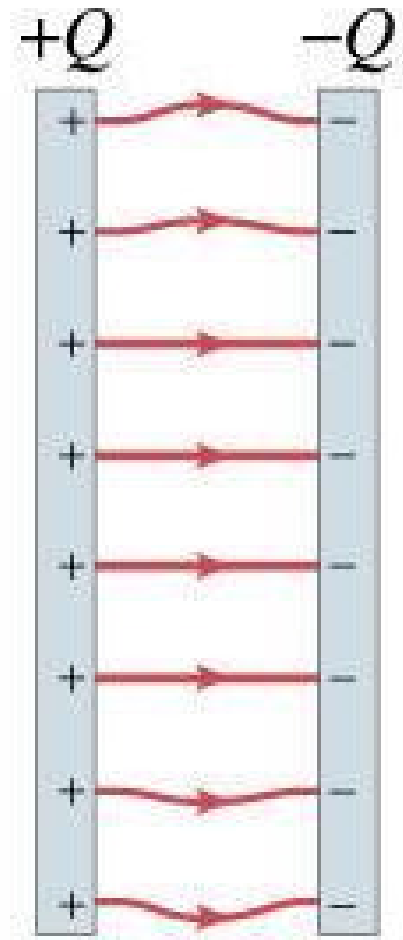






The Electric Field

The electric field between two closely spaced, oppositely charged parallel plates is constant.



Field Lines

Summary of field lines:

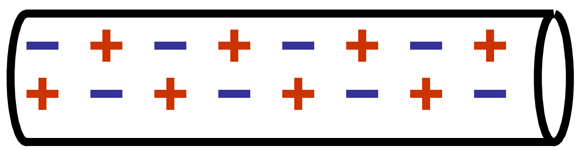
Field lines start on positive charges and end on negative charges; the number is proportional to the magnitude of the charge.

The magnitude of the field is proportional to the density of the lines.

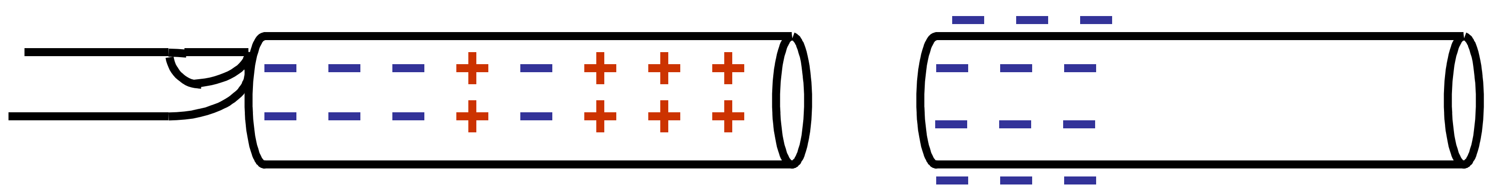
Field lines indicate the direction of the field; the field is tangent to the line.

Electric Fields and Conductors

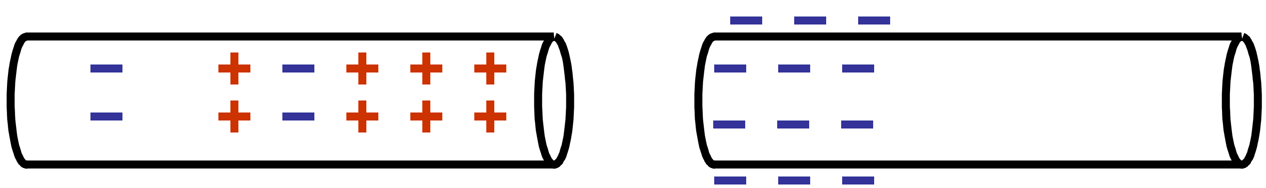
Charging by induction:



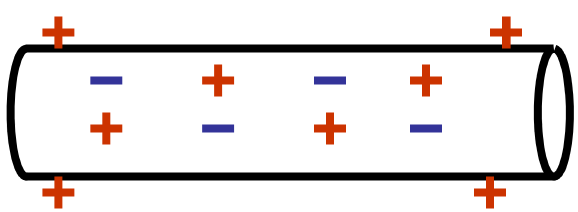
Electrically neutral



When finger is removed

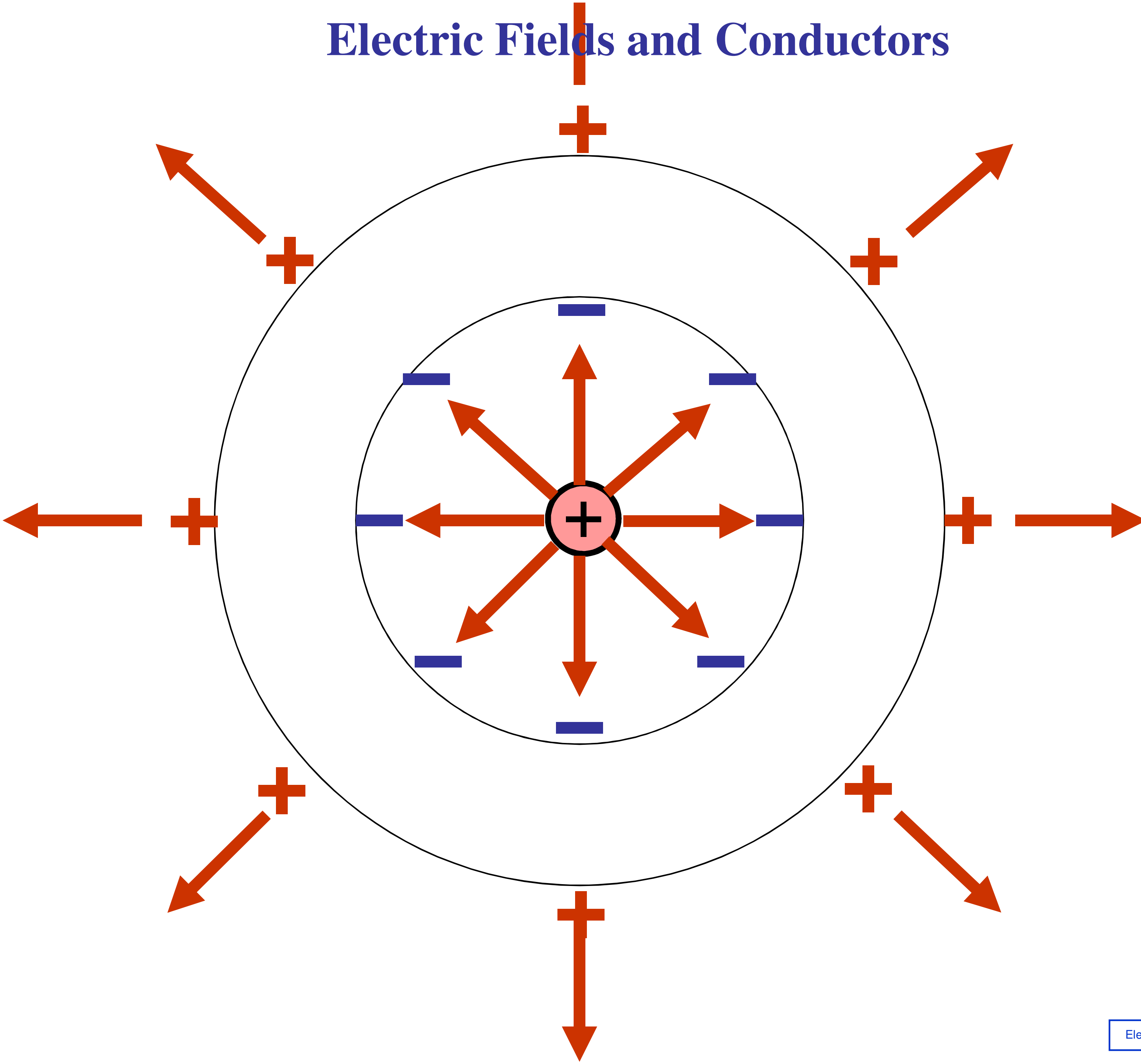


When charged rod is removed

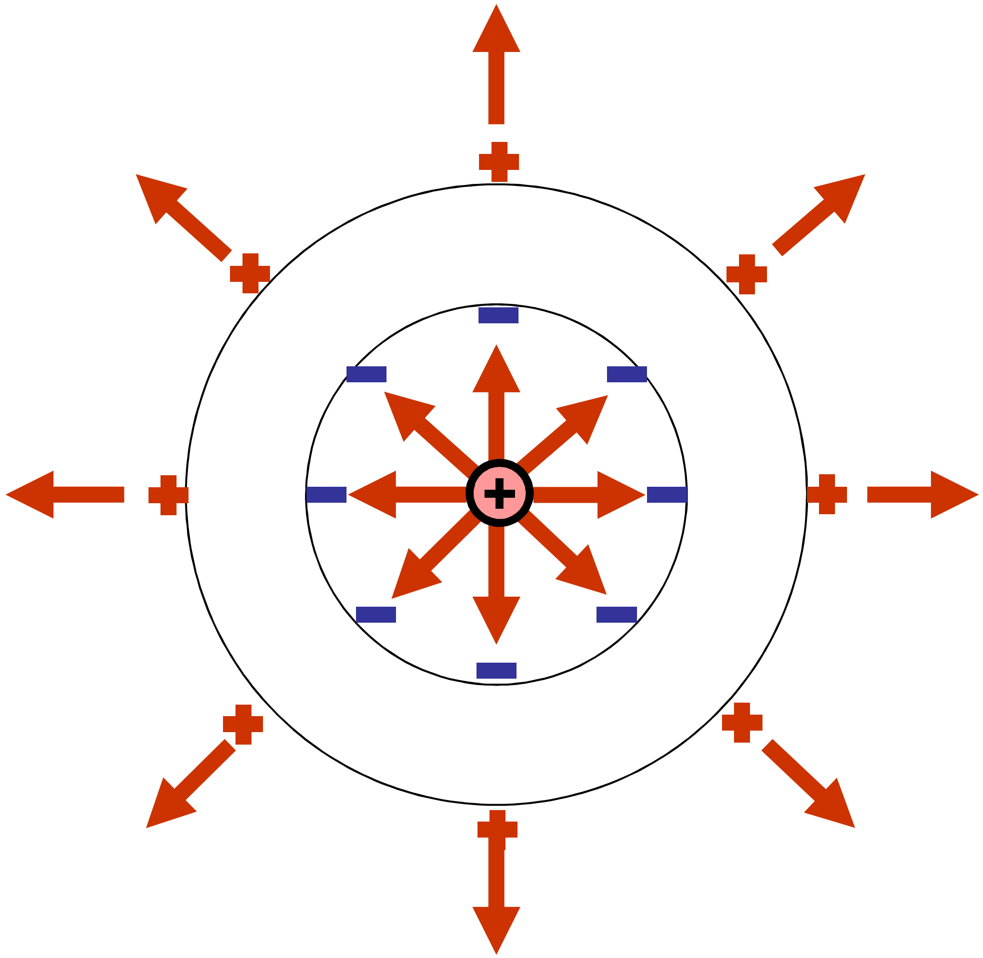


Net positive charge
(on outer surface)

Electric Fields and Conductors



Electric Fields and Conductors



The electric field is perpendicular to the surface of a conductor

The electric field is zero inside a conductor

Summary

Two kinds of electric charge – positive and negative

Charge is conserved

Charge on electron: $e = 1.602 \times 10^{-19}$ C

Conductors: electrons free to move

Insulators: nonconductors

Charge is quantized in units of *e*

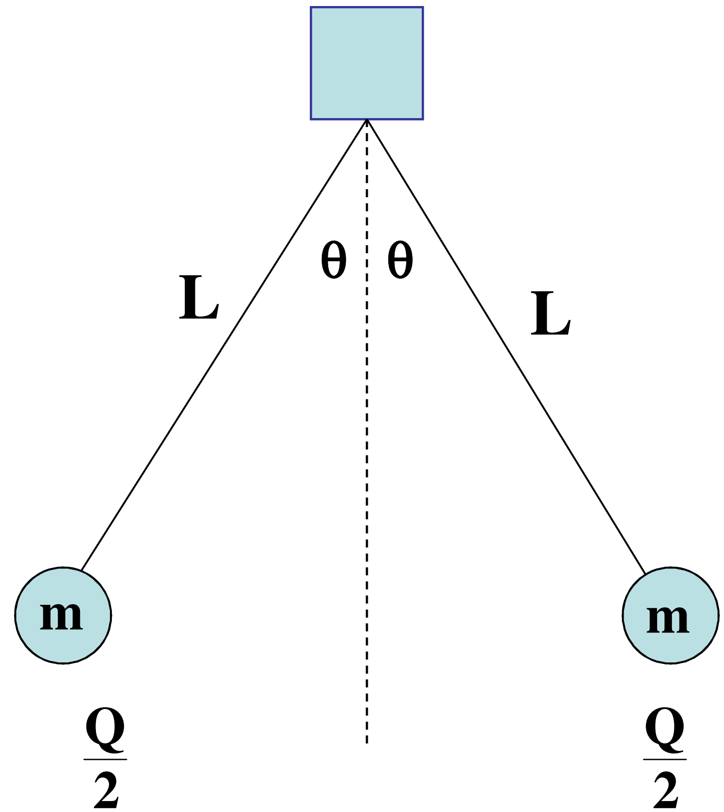
Objects can be charged by conduction or induction

Coulomb's law: $F = k\frac{Q_1 Q_1}{r^2}$

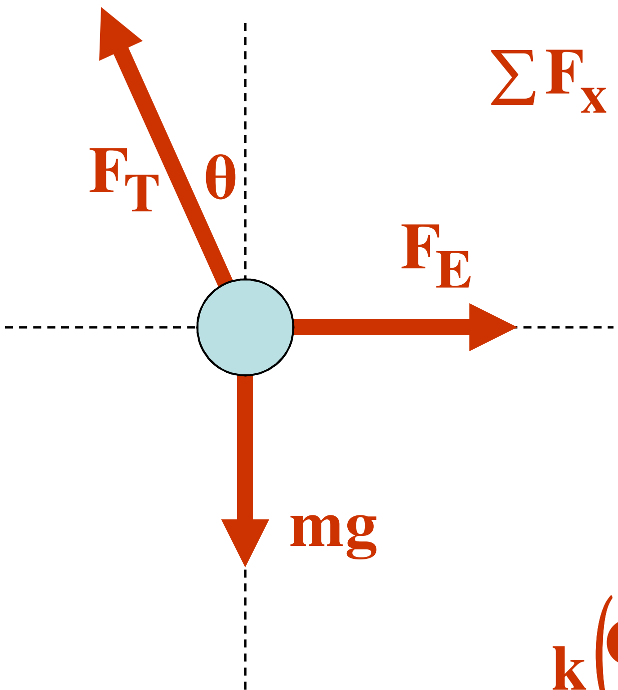
Electric field is force per unit charge: $E = \frac{F}{q}$

Problem 16-64

A large electroscope is made with "leaves" that are 78 cm long wires with tiny 24 g spheres at the ends. When charged, nearly all the charge resides on the spheres. If the wires each make a 30° angle with the vertical what total charge Q must have been applied to the electroscope? Ignore the mass of the wires.



Problem 16-64



$$\sum F_x = F_E - F_T \sin\theta = 0 \qquad \sum F_y = F_T \cos\theta - mg = 0$$

$$F_T \sin\theta = F_E \qquad F_T \cos\theta = mg$$

$$\frac{F_T \sin\theta = F_E}{F_T \cos\theta = mg} \quad \Rightarrow \quad F_E = mg\tan\theta$$

$$F_E = \frac{k\left(Q/2\right)^2}{d^2} = mg\tan\theta \quad \Rightarrow \quad Q = 2d\sqrt{\frac{mg\tan\theta}{k}}$$

$$Q = 2(0.78\,\text{m})\sqrt{\frac{(0.024\,\text{kg})(\tan 30^o)}{8.988 \times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2}} \qquad \boxed{= 6.1 \times 10^{-6}\ \text{C}}$$


END
325
575
50
300