- Fluids at Rest

HOW MANY PEOPLE THIS LIFE RAFT HOLD?

Fluids at Rest

HOT AIR BALLOONS use heated air, which is less dense than the surrounding air, to create an upward buoyant force. According to Archi-medes' Principle, the buoyant force is equal to the weight of the air displaced by the balloon.

Objectives: After completing this module, you should be able to:

- Define and apply the concepts of density and fluid pressure to solve physical problems.
- Define and apply concepts of absolute, gauge, and atmospheric pressures.
- State Pascal's law and apply for input and output pressures.
- State and apply Archimedes' Principle to solve physical problems.

Mass Density

$$Density = \frac{mass}{volume}; \quad \rho = \frac{m}{V}$$

Lead: 11,300 kg/m^3

Wood: 500 kg/m^3

Wood

2 kg, 4000 cm^3

Lead

4000 cm^3

45.2 kg

Same volume

Lead

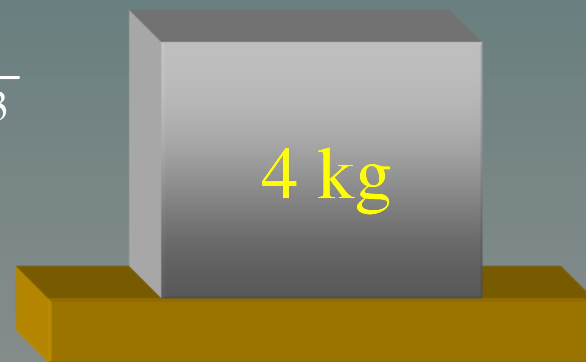
177 cm^3

2 kg

Same mass

Example 1: The density of steel is 7800 kg/m^3. What is the volume of a 4-kg block of steel?

$$\rho = \frac{m}{V}; \qquad V = \frac{m}{\rho} = \frac{4\text{ kg}}{7800\text{ kg/m}^3}$$



$$V = 5.13 \times 10^{-4}\text{ m}^3$$

What is the mass if the volume is 0.046 m^3?

$$m = \rho V = (7800\text{ kg/m}^3)(0.046\text{ m}^3);$$

$$m = 359\text{ kg}$$

Relative Density

The relative density ρ_r of a material is the ratio of its density to the density of water (1000 kg/m^3).

$$\rho_r = \frac{\rho_x}{1000\ \text{kg/m}^3}$$

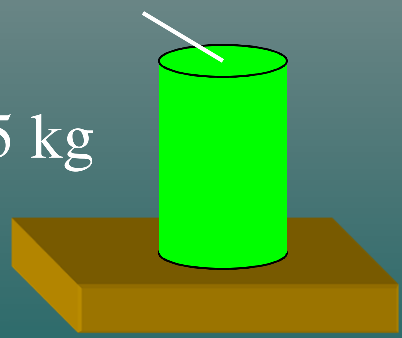
Examples:

Steel (7800 kg/m^3)	$\rho_r = 7.80$
Brass (8700 kg/m^3)	$\rho_r = 8.70$
Wood (500 kg/m^3)	$\rho_r = 0.500$

Pressure

Pressure is the ratio of a force F to the area A over which it is applied:

$$\text{Pressure} = \frac{Force}{Area}; \qquad P = \frac{F}{A}$$



$$P = \frac{F}{A} = \frac{(1.5\text{ kg})(9.8\text{ m/s}^2)}{2 \times 10^{-4}\text{ m}^2}$$

$$P = 73{,}500\text{ N/m}^2$$

The Unit of Pressure (Pascal):

A pressure of one pascal (1 Pa) is defined as a force of one newton (1 N) applied to an area of one square meter ($1\ m^2$).

$$\text{Pascal:} \quad 1\ \text{Pa} = 1\ \text{N/m}^2$$

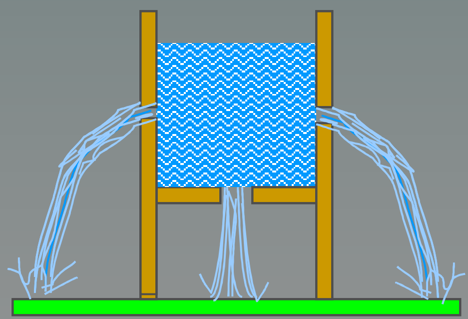
In the previous example the pressure was $73{,}500\ \text{N/m}^2$. This should be expressed as:

$$P = 73{,}500\ \text{Pa}$$

Fluid Pressure

A liquid or gas cannot sustain a shearing stress - it is only restrained by a boundary. Thus, it will exert a force against and perpendicular to that boundary.

- The force F exerted by a fluid on the walls of its container always acts perpendicular to the walls.

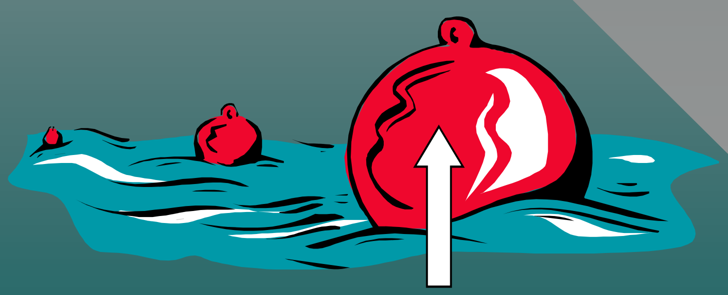


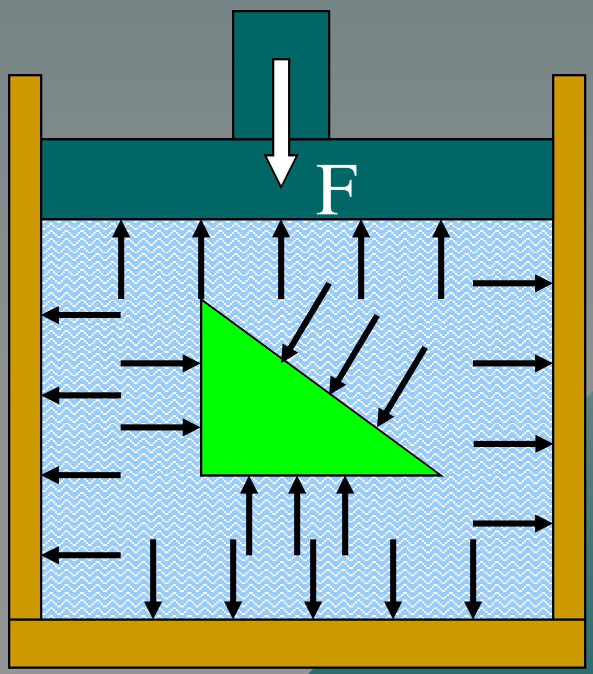
Water flow shows ⊥ F

Fluid Pressure

Fluid exerts forces in many directions. Try to submerse a rubber ball in water to see that an upward force acts on the float.

- Fluids exert pressure in all directions.



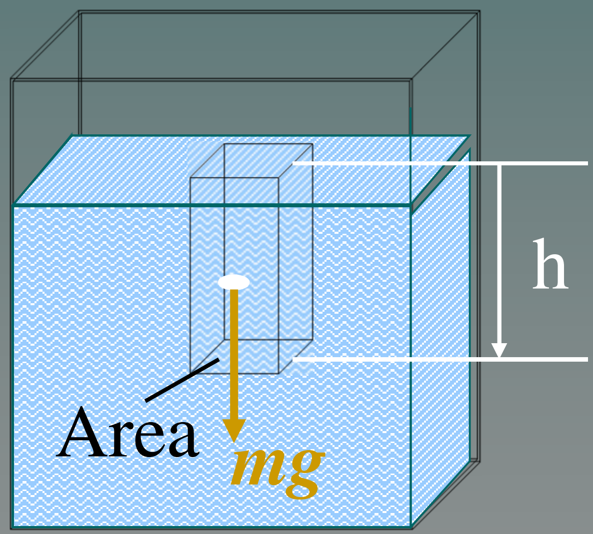


Pressure vs. Depth in Fluid

Pressure = force/area

$$P = \frac{mg}{A}; \quad m = \rho V; \quad V = Ah$$

$$P = \frac{\rho V g}{A} = \frac{\rho A h g}{A}$$



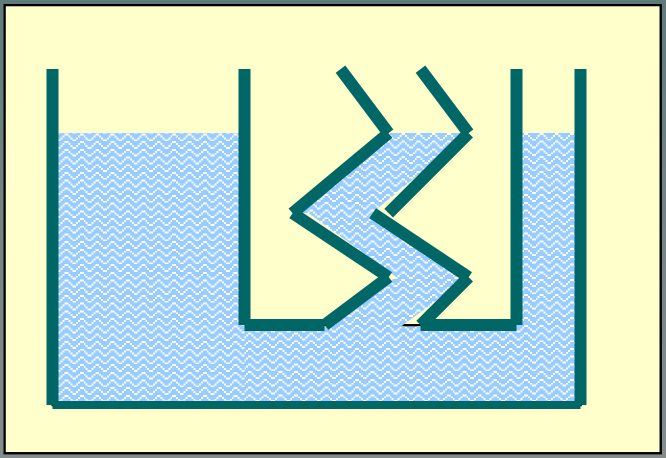
- Pressure at any point in a fluid is directly proportional to the density of the fluid and to the depth in the fluid.

Fluid Pressure:

$$P = \rho g h$$

Independence of Shape and Area.

Water seeks its own level, indicating that fluid pressure is independent of area and shape of its container.



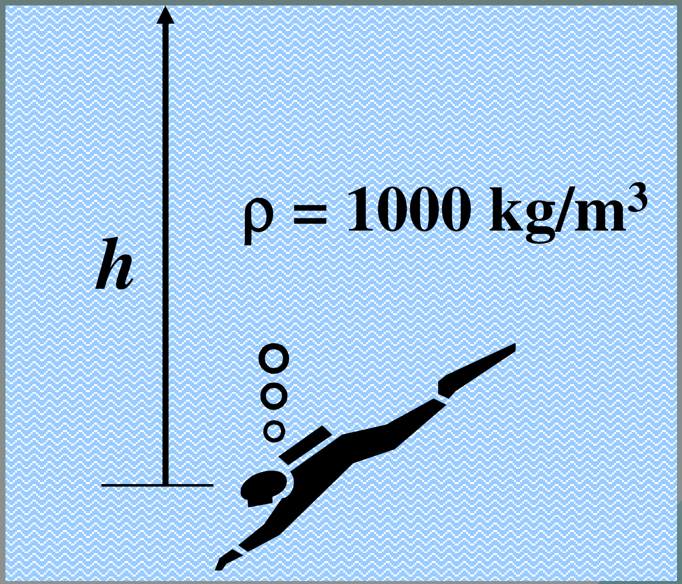
- At any depth h below the surface of the water in any column, the pressure P is the same. The shape and area are not factors.

Properties of Fluid Pressure

- The forces exerted by a fluid on the walls of its container are always perpendicular.
- The fluid pressure is directly proportional to the depth of the fluid and to its density.
- At any particular depth, the fluid pressure is the same in all directions.
- Fluid pressure is independent of the shape or area of its container.

Example 2. A diver is located 20 m below the surface of a lake (ρ = 1000 kg/m^3). What is the pressure due to the water?

The difference in pressure from the top of the lake to the diver is:



$$\Delta P = \rho g h$$

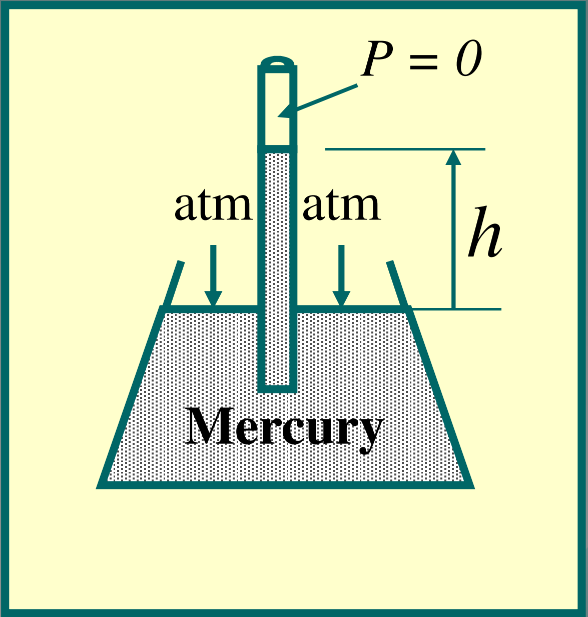
h = 20 m; g = 9.8 m/s^2

$$\Delta P = (1000 \text{ kg/m}^3)(9.8 \text{ m/s}^2)(20 \text{ m})$$

$$\boxed{\Delta P = 196 \text{ kPa}}$$

Atmospheric Pressure

One way to measure atmospheric pressure is to fill a test tube with mercury, then invert it into a bowl of mercury.



Density of Hg = 13,600 kg/m^3

$P_{atm} = \rho g h$ $\qquad$ $h = 0.760$ m

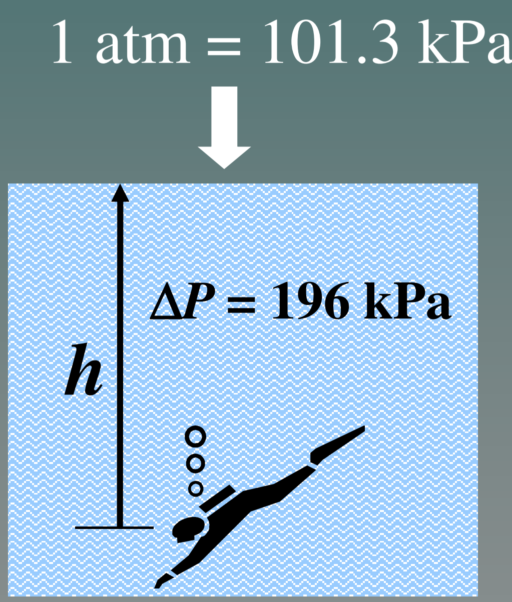
$$P_{atm} = (13{,}600 \text{ kg/m}^3)(9.8 \text{ m/s}^2)(0.760 \text{ m})$$

$$P_{atm} = 101{,}300 \text{ Pa}$$

Absolute Pressure

Absolute Pressure: The sum of the pressure due to a fluid and the pressure due to atmosphere.

Gauge Pressure: The difference between the absolute pressure and the pressure due to the atmosphere:



Absolute Pressure = Gauge Pressure + 1 atm

ΔP = 196 kPa

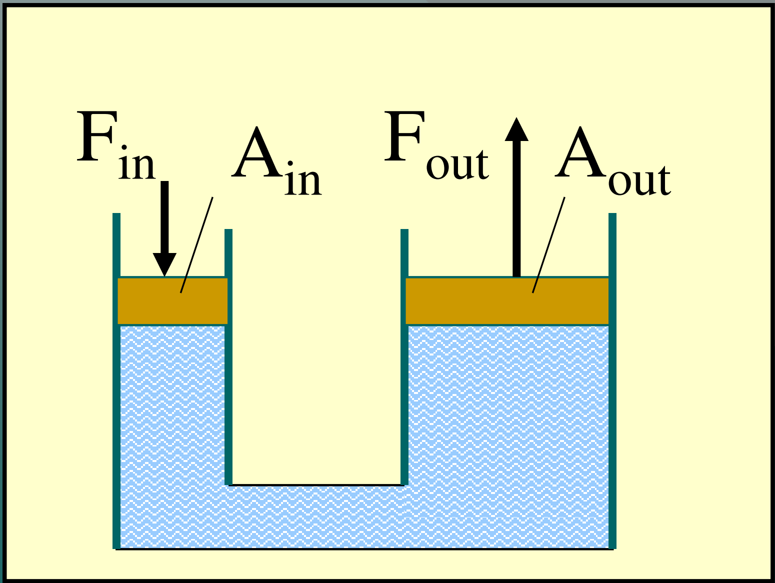
1 atm = 101.3 kPa

P_{abs} = 196 kPa + 101.3 kPa

P_{abs} = 297 kPa

Pascal's Law

Pascal's Law: An external pressure applied to an enclosed fluid is transmitted uniformly throughout the volume of the liquid.



Pressure in = Pressure out

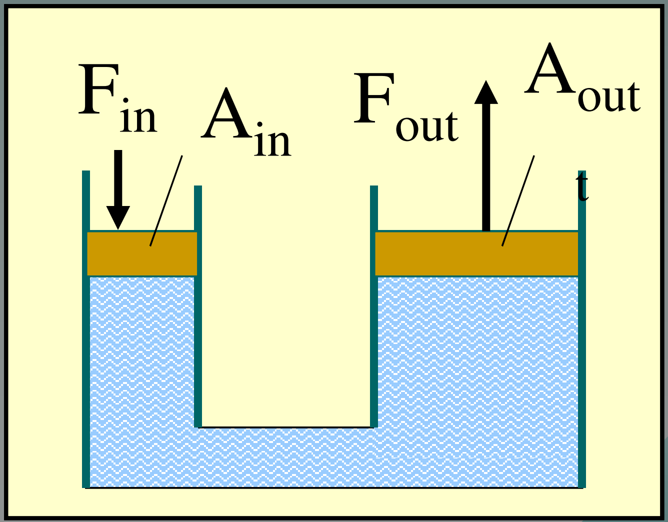
$$\frac{F_{in}}{A_{in}} = \frac{F_{out}}{A_{out}}$$

Example 3. The smaller and larger pistons of a hydraulic press have diameters of 4 cm and 12 cm. What input force is required to lift a 4000 N weight with the output piston?

$$\frac{F_{in}}{A_{in}} = \frac{F_{out}}{A_{out}}; \qquad F_{in} = \frac{F_{out} A_{in}}{A_{out}}$$

$$R = \frac{D}{2}; \qquad Area = \pi R^2$$

$$F_{in} = \frac{(4000\text{ N})(\pi)(2\text{ cm})^2}{\pi(6\text{ cm})^2}$$

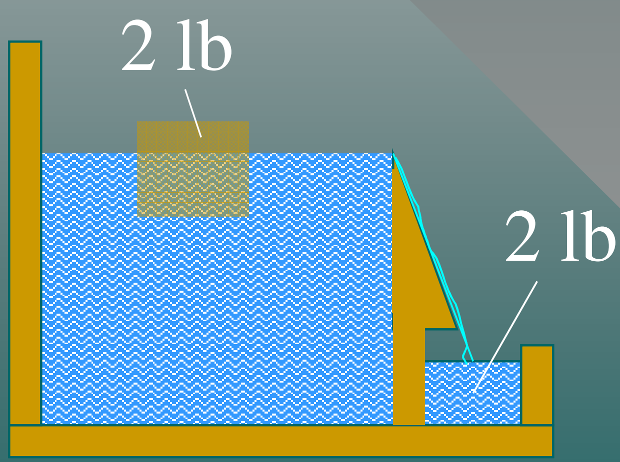


R_{in}= 2 cm; R = 6 cm

$$F = 444\text{ N}$$

Archimedes' Principle

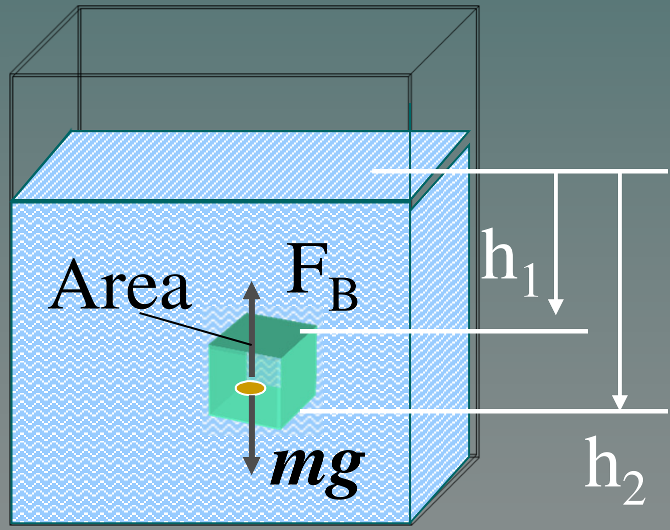
- An object that is completely or partially submerged in a fluid experiences an upward **buoyant force** equal to the weight of the fluid displaced.



The buoyant force is due to the displaced fluid. The block material doesn't matter.

Calculating Buoyant Force

The buoyant force F_B is due to the difference of pressure ΔP between the top and bottom surfaces of the submerged block.



$$\Delta P = \frac{F_B}{A} = P_2 - P_1; \quad F_B = A(P_2 - P_1)$$

$$F_B = A(P_2 - P_1) = A(\rho_f g h_2 - \rho_f g h_1)$$

$$F_B = (\rho_f g) A(h_2 - h_1); \quad V_f = A(h_2 - h_1)$$

V_f is volume of fluid displaced.

Buoyant Force:

$$F_B = \rho_f g V_f$$

Example 4: A 2-kg brass block is attached to a string and submerged underwater. Find the buoyant force and the tension in the rope.

All forces are balanced:

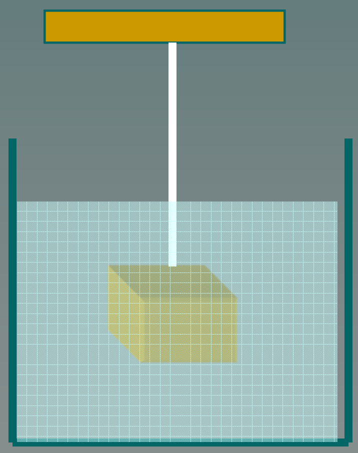
$$F_B + T = mg \qquad F_B = \rho_w g V_w$$

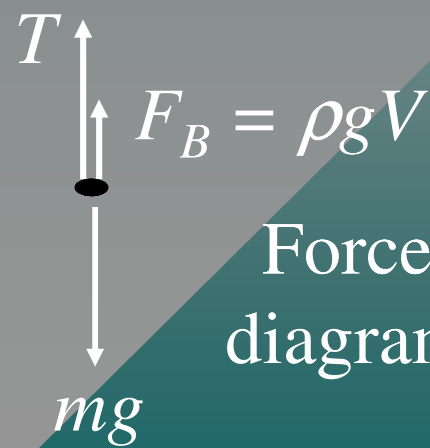
$$\rho_b = \frac{m_b}{V_b}; \quad V_b = \frac{m_b}{\rho_b} = \frac{2\text{ kg}}{8700\text{ kg/m}^3}$$

$$V_b = V_w = 2.30 \times 10^{-4}\text{ m}^3$$

$$F_b = (1000\text{ kg/m}^3)(9.8\text{ m/s}^2)(2.3 \times 10^{-4}\text{ m}^3)$$

$$F_B = 2.25\text{ N}$$

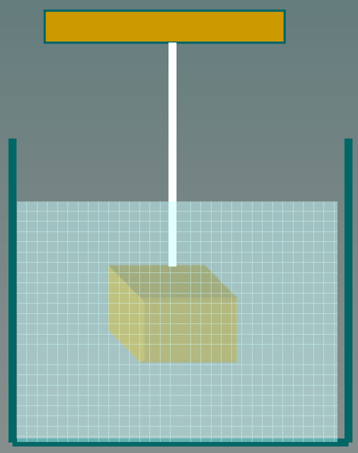




Force diagram

Example 4 (Cont.): A 2-kg brass block is attached to a string and submerged underwater. Now find the the tension in the rope.

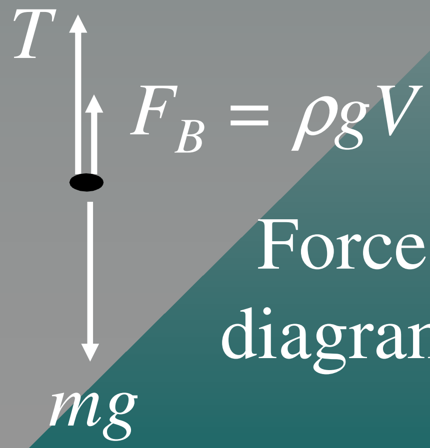
$$F_B = 2.25 \text{ N}$$



$$F_B + T = mg \qquad T = mg - F_B$$

$$T = (2 \text{ kg})(9.8 \text{ m/s}^2) - 2.25 \text{ N}$$

$$T = 19.6 \text{ N} - 2.25 \text{ N}$$



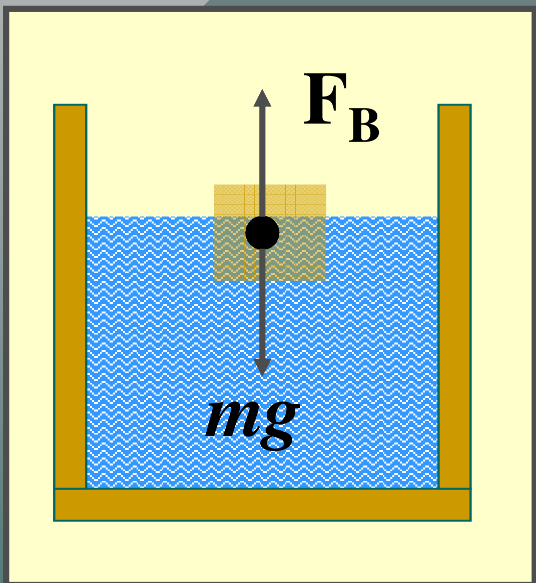
Force diagram

$$T = 17.3 \text{ N}$$

This force is sometimes referred to as the apparent weight.

Floating objects:

When an object floats, partially submerged, the buoyant force exactly balances the weight of the object.



$$F_B = \rho_f g V_f \qquad m_x g = \rho_x V_x g$$

$$\rho_f g V_f = \rho_x V_x g$$

Floating Objects: $\rho_f V_f = \rho_x V_x$

If V_f is volume of displaced water V_{wd}, the relative density of an object x is given by:

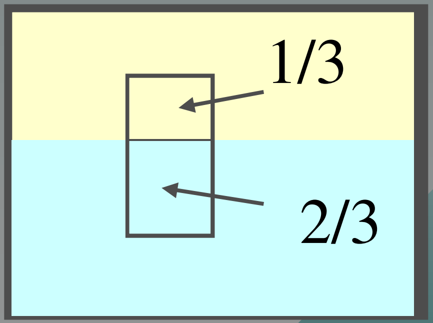
Relative Density:

$$\rho_r = \frac{\rho_x}{\rho_w} = \frac{V_{wd}}{V_x}$$

Example 5: A student floats in a salt lake with one-third of his body above the surface. If the density of his body is 970 kg/m^3, what is the density of the lake water?

Assume the student's volume is 3 m^3.

$V_s = 3\text{ m}^3;\ \ V_{wd} = 2\text{ m}^3;\ \ \rho_s = 970\text{ kg/m}^3$



$$\rho_w V_{wd} = \rho_s V_s$$

$$\frac{\rho_s}{\rho_w} = \frac{V_{wd}}{V_s} = \frac{2\text{ m}^3}{3\text{ m}^3}; \quad \rho_w = \frac{3\rho_s}{2}$$

$$\rho_w = \frac{3\rho_s}{2} = \frac{3(970\text{ kg/m}^3)}{2}$$

$$\rho_w = 1460\text{ kg/m}^3$$

Problem Solving Strategy

1. Draw a figure. Identify givens and what is to be found. Use consistent units for P, V, A, and ρ.

2. Use absolute pressure P_{abs} unless problem involves a difference of pressure ΔP.

3. The difference in pressure ΔP is determined by the density and depth of the fluid:

$$P_2 - P_1 = \rho g h; \quad \rho = \frac{m}{V}; \quad P = \frac{F}{A}$$

Problem Strategy (Cont.)

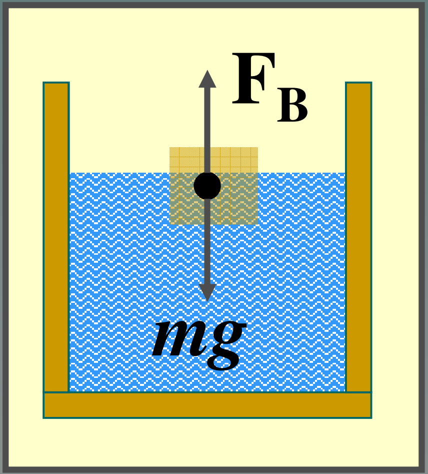
4. Archimedes' Principle: A submerged or floating object experiences an buoyant force equal to the weight of the displaced fluid:

$$F_B = m_f g = \rho_f g V_f$$

5. Remember: *m, r* and *V* refer to the *displaced fluid*. The buoyant force has nothing to do with the mass or density of the object in the fluid. (If the object is completely submerged, then its volume is equal to that of the fluid displaced.)

Problem Strategy (Cont.)

6. For a floating object, F_B is equal to the weight of that object; i.e., the weight of the object is equal to the weight of the displaced fluid:



$$m_x g = m_f g \quad \text{or} \quad \rho_x V_x = \rho_f V_f$$

Summary

$$Density = \frac{mass}{volume}; \quad \rho = \frac{m}{V}$$

$$\rho_r = \frac{\rho_x}{1000\ \text{kg/m}^3}$$

$$\text{Pressure} = \frac{Force}{Area}; \quad P = \frac{F}{A}$$

<u>Fluid Pressure:</u>

$$P = \rho g h$$

Pascal: $1\ \text{Pa} = 1\ \text{N/m}^2$

Summary (Cont.)

Pascal's Law:

$$\frac{F_{in}}{A_{in}} = \frac{F_{out}}{A_{out}}$$

Archimedes' Principle:

Buoyant Force:

$$F_B = \rho_f g V_f$$

CONCLUSION:
Fluids at Rest