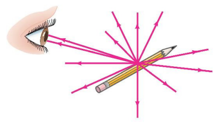
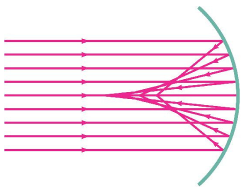
Geometric Optics





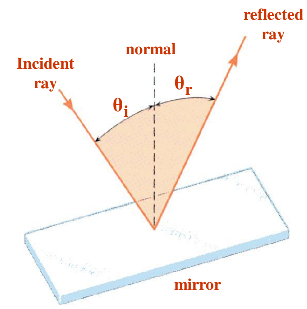
Physics 2054
Lecture Notes

Reflection

Law of Reflection

The incident ray, the reflected ray, and the normal to the surface all lie in the same plane, and the angle of reflection θ_r equals the angle of incidence θ_i

$$\theta_i = \theta_r$$

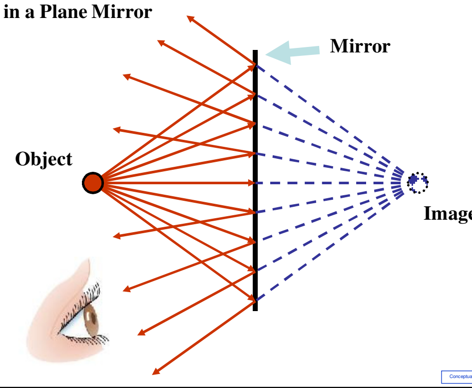


Topics

- The Ray Model of Light
- Reflection
- Formation of Images by Spherical Mirrors
- Index of Refraction
- Refraction
- Total Internal Reflection
- Thin Lenses

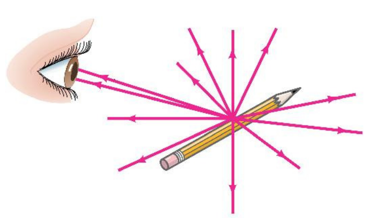
Reflection

Image in a Plane Mirror



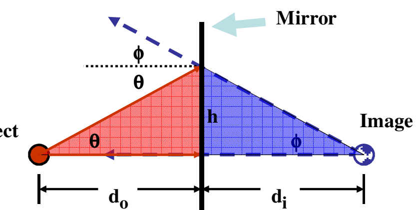
The Ray Model of Light

Light very often travels in straight lines. We represent light using rays, which are straight lines emanating from an object. This is an idealization, but is very useful for geometric optics.



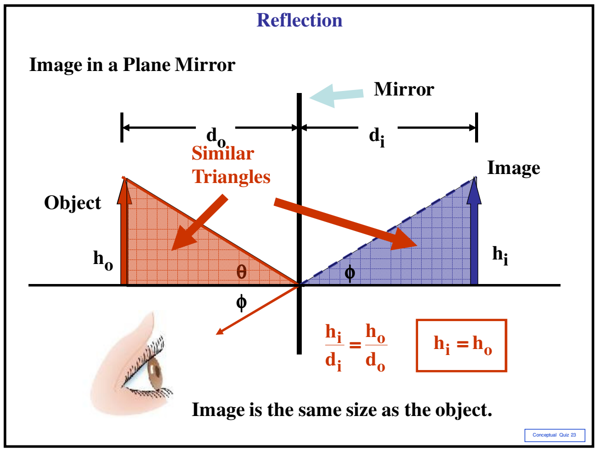
Reflection

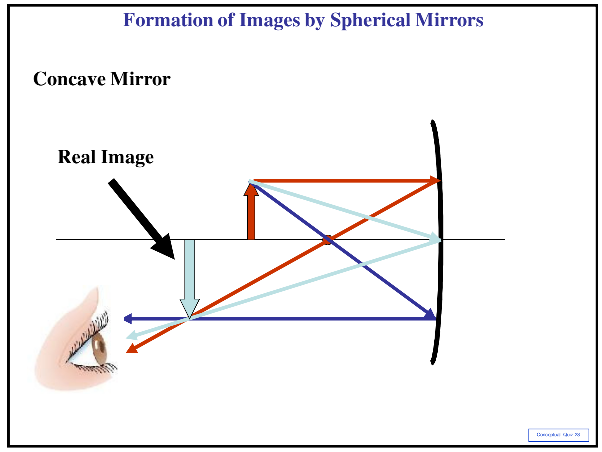
Image in a Plane Mirror

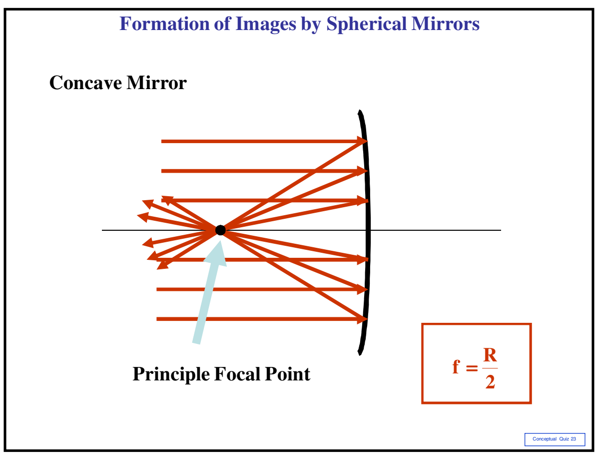


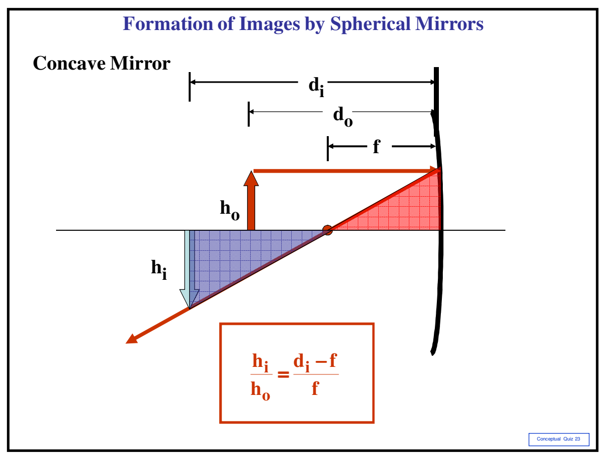
$$\frac{d_i}{h} = \frac{d_o}{h} \Rightarrow d_i = d_o$$

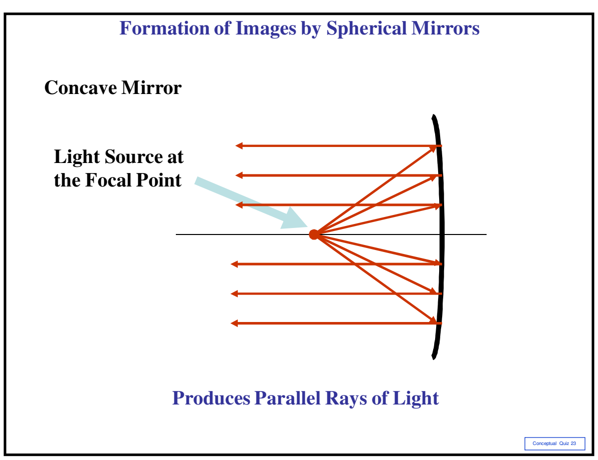
The image is as far behind the mirror as the object is in front of the mirror.

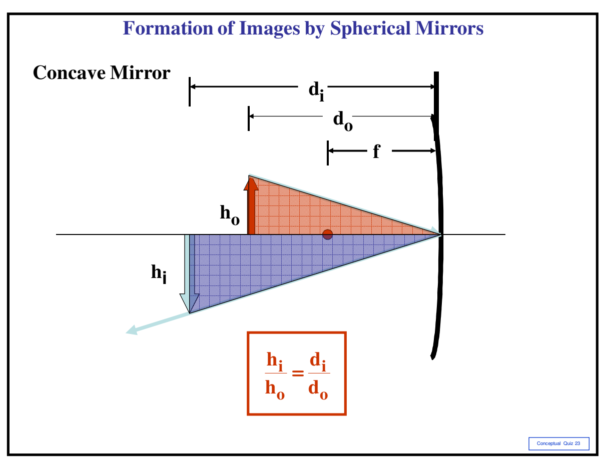

Reflection
Image in a Plane Mirror
Mirror
d_o
d_i
Similar Triangles
Image
Object
h_o
h_i
θ
ϕ
ϕ
$\frac{h_i}{d_i} = \frac{h_o}{d_o}$
$h_i = h_o$
Image is the same size as the object.


Formation of Images by Spherical Mirrors
Concave Mirror
Real Image


Formation of Images by Spherical Mirrors
Concave Mirror
Principle Focal Point
$f = \frac{R}{2}$


Formation of Images by Spherical Mirrors
Concave Mirror
d_i
d_o
f
h_o
h_i
$\frac{h_i}{h_o} = \frac{d_i - f}{f}$


Formation of Images by Spherical Mirrors
Concave Mirror
Light Source at the Focal Point
Produces Parallel Rays of Light


Formation of Images by Spherical Mirrors
Concave Mirror
d_i
d_o
f
h_o
h_i
$\frac{h_i}{h_o} = \frac{d_i}{d_o}$

Formation of Images by Spherical Mirrors

$$\frac{h_i}{h_o} = \frac{d_i - f}{f} \qquad \frac{h_i}{h_o} = \frac{d_i}{d_o}$$

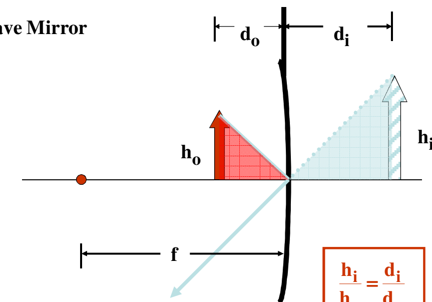
$$\frac{d_i - f}{f} = \frac{d_i}{d_o}$$

$$\boxed{\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i}} \quad \Rightarrow \quad f = \frac{d_o d_i}{d_o + d_i}$$

Mirror Equation

Formation of Images by Spherical Mirrors

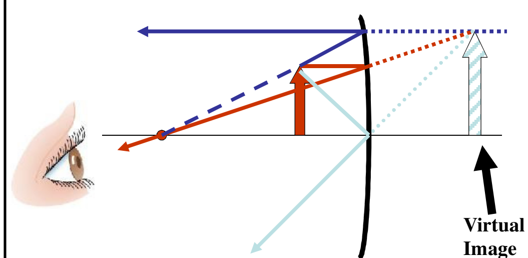
Concave Mirror



$$\boxed{\frac{h_i}{h_o} = \frac{d_i}{d_o}}$$

Formation of Images by Spherical Mirrors

Concave Mirror



Formation of Images by Spherical Mirrors

$$\frac{h_i}{h_o} = \frac{f + d_i}{f} \qquad \frac{h_i}{h_o} = \frac{d_i}{d_o}$$

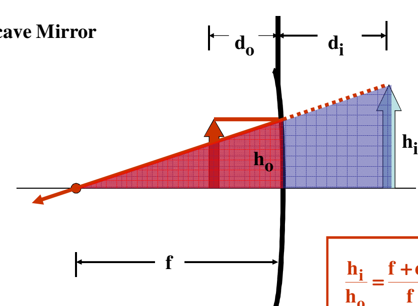
$$\frac{f + d_i}{f} = \frac{d_i}{d_o}$$

$$\frac{1}{f} = \frac{1}{d_o} - \frac{1}{d_i}$$

$$\boxed{\frac{1}{f} = \frac{1}{d_o} + \frac{1}{(-d_i)}}$$

Formation of Images by Spherical Mirrors

Concave Mirror



$$\boxed{\frac{h_i}{h_o} = \frac{f + d_i}{f}}$$

Formation of Images by Spherical Mirrors

Problem 23-10

A mirror at an amusement park shows an upright image of any person who stands 1.4 m in front of it. If the image is three times the person's height, what is the radius of curvature?

$$M = \frac{h_i}{h_o} = -\frac{d_i}{d_o} \qquad \frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i} \quad \text{or} \quad f = \frac{d_o d_i}{d_o + d_i}$$

$$d_i = M d_o = 3(1.4\,\text{m})$$

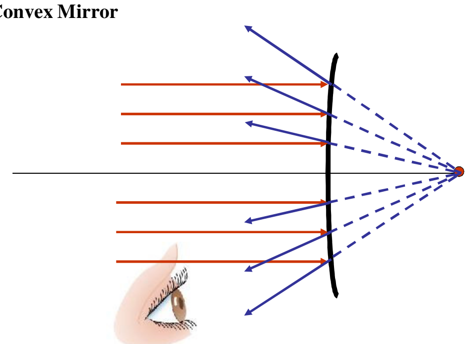
$$d_i = 4.2\,\text{m}$$

$$f = \frac{1.4\,\text{m}(-4.2\,\text{m})}{1.4\,\text{m} + (-4.2\,\text{m})} = 2.1\,\text{m}$$

$$r = 2f = 2(2.1\,\text{m}) \quad \boxed{= 4.2\,\text{m}}$$

Formation of Images by Spherical Mirrors

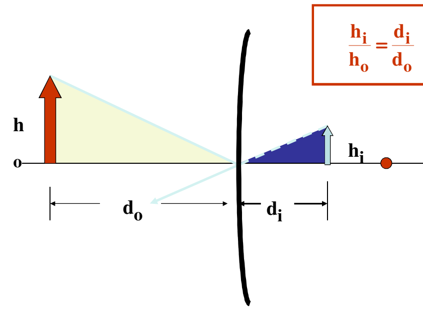
Convex Mirror



Formation of Images by Spherical Mirrors

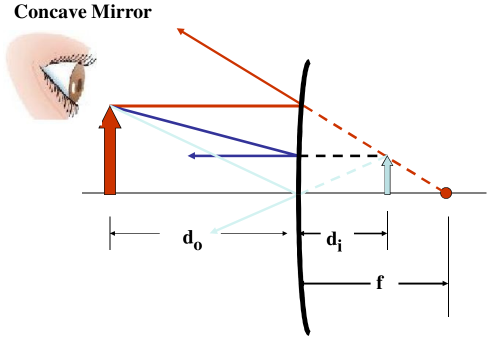
Concave Mirror

$$\frac{h_i}{h_o} = \frac{d_i}{d_o}$$



Formation of Images by Spherical Mirrors

Concave Mirror



Formation of Images by Spherical Mirrors

$$\frac{h_i}{h_o} = \frac{f - d_i}{f} \qquad \frac{h_i}{h_o} = \frac{d_i}{d_o}$$

$$\frac{f - d_i}{f} = \frac{d_i}{d_o}$$

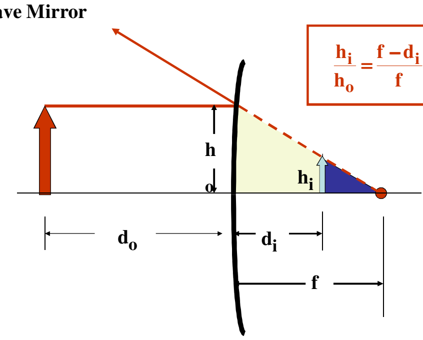
$$\frac{1}{d_i} - \frac{1}{f} = \frac{1}{d_o}$$

$$\boxed{\frac{1}{(-f)} = \frac{1}{d_o} + \frac{1}{(-d_i)}}$$

Formation of Images by Spherical Mirrors

Concave Mirror

$$\frac{h_i}{h_o} = \frac{f - d_i}{f}$$



Formation of Images by Spherical Mirrors

Problem 23-15

The image of a distant tree is virtual and very small when viewed in a curved mirror. The image appears to be 18 cm behind the mirror. What kind of mirror is it, and what is its radius of curvature?

$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{\infty} + \frac{1}{-18\,\text{cm}} \qquad f = -18\,\text{cm}$$

$$r = 2f = 2(-18\,\text{cm}) \quad \boxed{= -36\,\text{cm}}$$

TABLE 23–1
Indices of Refraction†

Medium	$n = c/v$
Vacuum	1.0000
Air (at STP)	1.0003
Water	1.33
Ethyl alcohol	1.36
Glass	
Fused quartz	1.46
Crown glass	1.52
Light flint	1.58
Lucite or Plexiglas	1.51
Sodium chloride	1.53
Diamond	2.42

† $\lambda = 589$ nm.

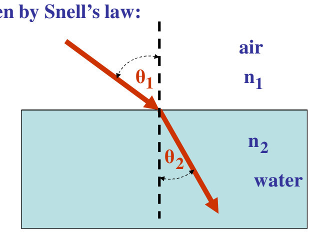
Index of Refraction

In general, light slows somewhat when traveling through a medium. The index of refraction of the medium is the ratio of the speed of light in vacuum to the speed of light in the medium:

$$n = \frac{c}{v}$$

Refraction

The angle of refraction depends on the indices of refraction, and is given by Snell's law:



$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

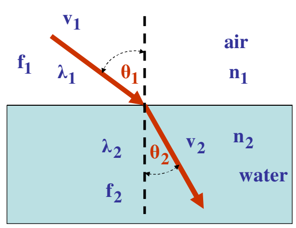
Index of Refraction

Problem 23-24

The speed of light in ice is 2.29×10^8 m/s. What is the index of refraction of ice?

$$n = \frac{c}{v} = \frac{3.00 \times 10^8 \text{ m/s}}{2.29 \times 10^8 \text{ m/s}} = 1.31$$

Refraction



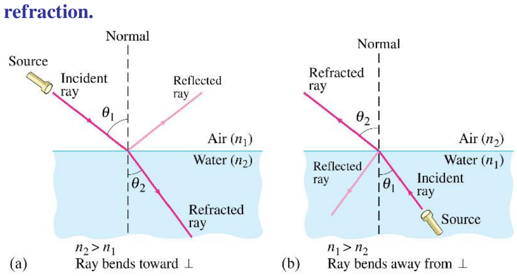
$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

$$\frac{\sin\theta_1}{\sin\theta_2} = \frac{n_2}{n_1} = \frac{v_1}{v_2} = \frac{\lambda_1}{\lambda_2}$$

$$f_1 = f_2$$

Refraction

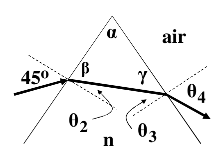
Light changes direction when crossing a boundary from one medium to another. This is called refraction, and the angle the outgoing ray makes with the normal is called the angle of refraction.



Index of Refraction

Problem 23-32

Light is incident on an equilateral glass prism at a 45.0° angle to one face. Calculate the angle at which light emerges from the opposite face. Assume that n = 1.58.



$$n_{air} \sin\theta_1 = n_2 \sin\theta_2$$

$$1.00 \sin 45^o = 1.58 \sin\theta_2 \quad \Rightarrow \theta_2 = 26.6^o$$

$$\alpha + \beta + \gamma = 180^o$$

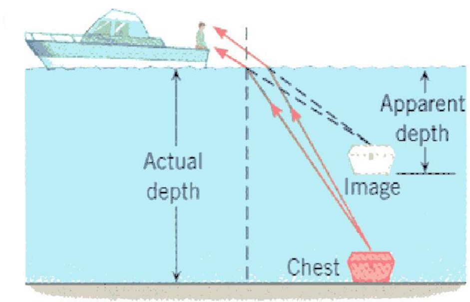
$$60^o + (90^o - 26.6^o) + (90^o - \theta_3) = 180^o \quad \Rightarrow \theta_3 = 33.6^o$$

$$n \sin\theta_3 = n_{air} \sin\theta_4$$

$$1.58 \sin 33.6^o = 1.00 \sin\theta_4 \quad \Rightarrow \theta_4 = 60.5^o$$

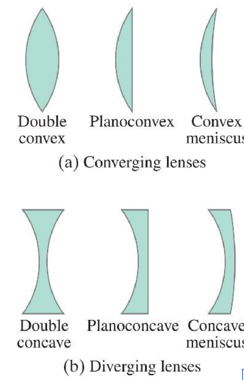
Refraction

Apparent Depth



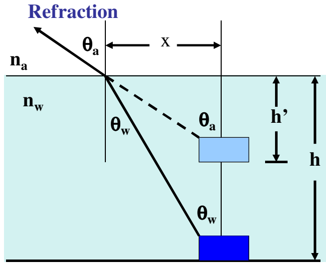
Thin Lenses

Thin lenses are those whose thickness is small compared to their radius of curvature. They may be either converging (a) or diverging (b).



Refraction

Apparent Depth



Snell's Law

$$n_a \sin\theta_a = n_w \sin\theta_w$$

$$\frac{\sin\theta_a}{\sin\theta_w} = \frac{n_w}{n_a}$$

From the diagram

$$\tan\theta_a = \frac{x}{h'}$$

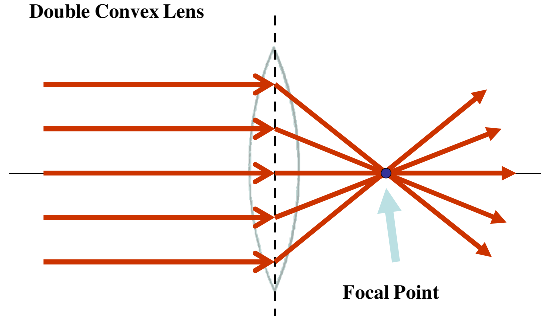
$$\tan\theta_w = \frac{x}{h}$$

$$\Rightarrow \frac{\tan\theta_a}{\tan\theta_w} = \frac{h}{h'} \approx \frac{n_w}{n_a}$$

$$\Rightarrow h' \approx h\left(\frac{n_a}{n_w}\right)$$

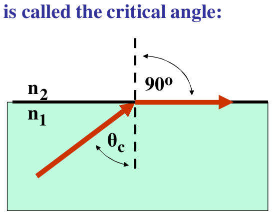
Thin Lenses

Double Convex Lens



Total Internal Reflection

If light passes into a medium with a smaller index of refraction, the angle of refraction is larger. There is an angle of incidence for which the angle of refraction will be 90°; this is called the critical angle:



Snell's Law

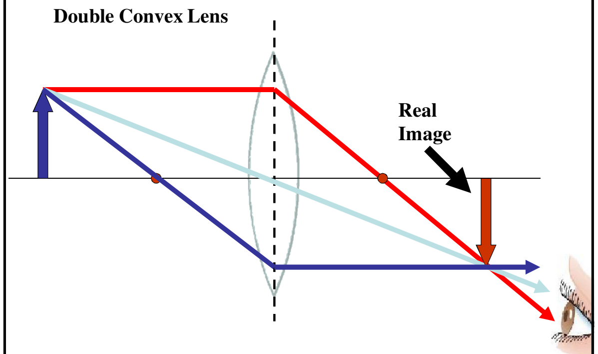
$$n_1 \sin\theta_c = n_2 \sin 90^o$$

$$\sin\theta_c = \frac{n_2 \sin 90^o}{n_1}$$

$$\sin\theta_c = \frac{n_2}{n_1}$$

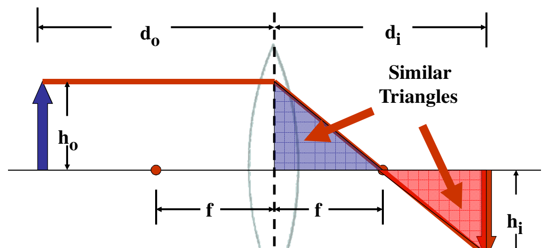
Thin Lenses

Double Convex Lens



Thin Lenses

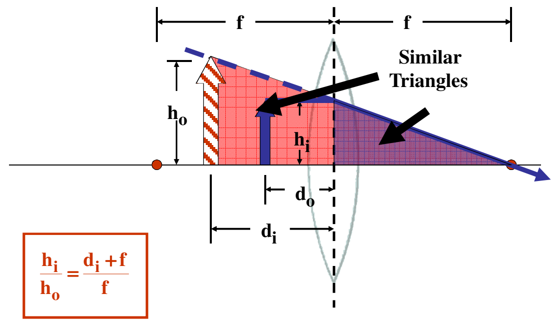
Double Convex Lens



$$\frac{h_i}{h_o} = \frac{d_i - f}{f}$$

Thin Lenses

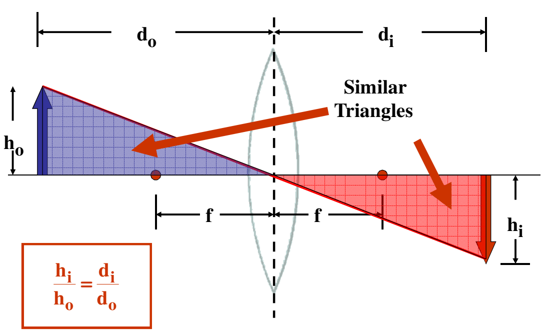
Convex Lens



$$\frac{h_i}{h_o} = \frac{d_i + f}{f}$$

Thin Lenses

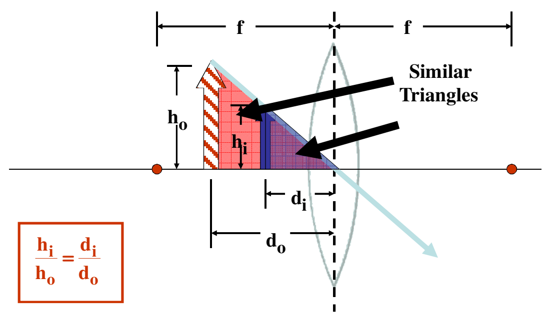
Convex Lens



$$\frac{h_i}{h_o} = \frac{d_i}{d_o}$$

Thin Lenses

Convex Lens



$$\frac{h_i}{h_o} = \frac{d_i}{d_o}$$

Thin Lenses

Double Convex Lens

$$\frac{h_i}{h_o} = \frac{d_i - f}{f} \qquad \frac{h_i}{h_o} = \frac{d_i}{d_o}$$

$$\frac{d_i - f}{f} = \frac{d_i}{d_o}$$

$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i}$$

Lens Equation

Thin Lenses

Convex Lens

$$\frac{h_i}{h_o} = \frac{d_i + f}{f} \qquad \frac{h_i}{h_o} = \frac{d_i}{d_o}$$

$$\frac{d_i + f}{f} = \frac{d_i}{d_o}$$

$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{(-d_i)}$$

Lens Equation

Formation of Images by Spherical Mirrors

Magnification:

$$M = \frac{h_i}{h_o} = -\frac{d_i}{d_o}$$

Power:

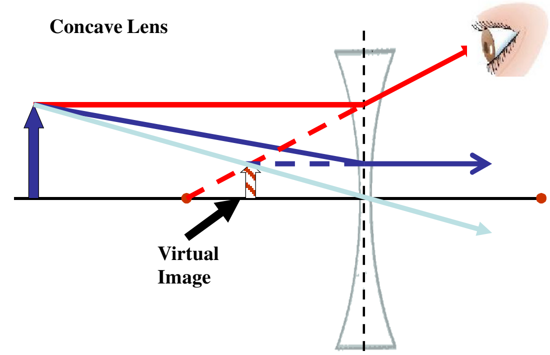
The unit for lens power is the diopter (D).

$$P = \frac{1}{f}$$

A 30 cm focal length lens has a power

$$P = \frac{1}{f} = \frac{1}{0.30\,\mathrm{m}} = 3.33\,\mathrm{D}$$

Thin Lenses



Index of Refraction

Problem 23-44

Sunlight is observed to focus at a point 18.5 cm behind a lens. What kind of lens is it?

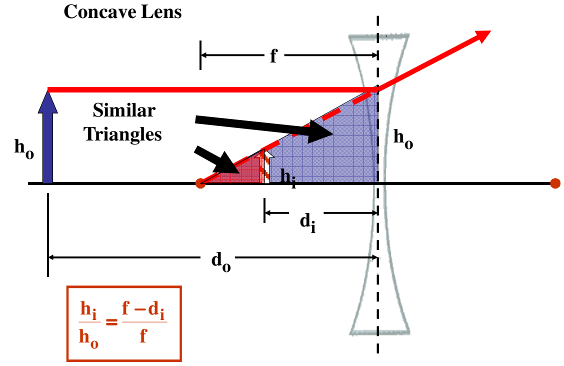
Converging Lens

What is its power in diopters?

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f} = P$$

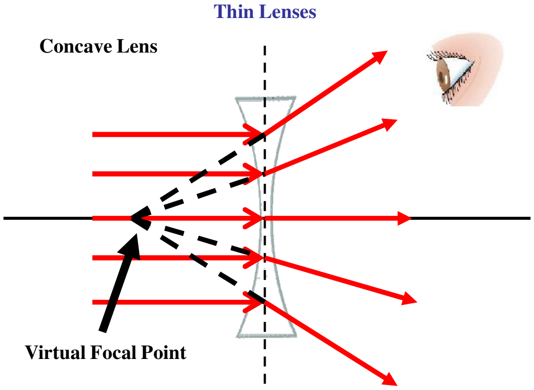
$$P = \frac{1}{\infty} + \frac{1}{0.185\,\mathrm{m}} = 5.41\,\mathrm{D}$$

Thin Lenses

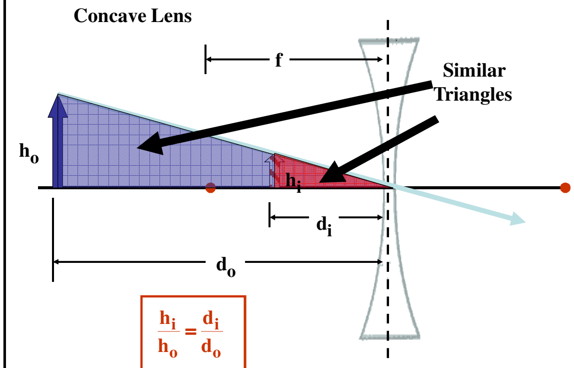


$$\frac{h_i}{h_o} = \frac{f - d_i}{f}$$

Thin Lenses



Thin Lenses



$$\frac{h_i}{h_o} = \frac{d_i}{d_o}$$

Thin Lenses

Concave Lens

$$\frac{h_i}{h_o} = \frac{f - d_i}{f}$$

$$\frac{h_i}{h_o} = \frac{d_i}{d_o}$$

$$\frac{f - d_i}{f} = \frac{d_i}{d_o}$$

$$\frac{1}{-f} = \frac{1}{d_o} + \frac{1}{(-d_i)}$$

Lens Equation

Conceptual Quiz 23