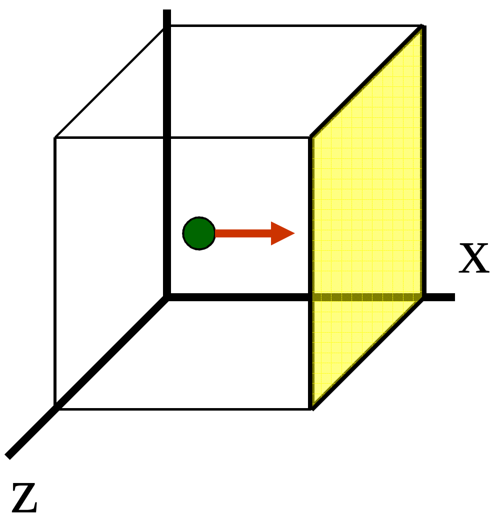
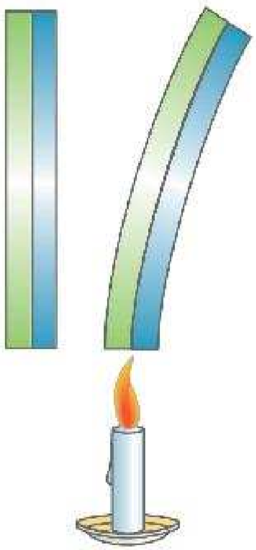
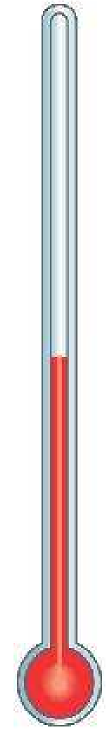
Temperature, Thermal Expansion and the Gas Laws







Physics 2053
Lecture Notes

Temperature,Thermal Expansion and the Gas Laws

Temperature and Thermometers

Thermal Equilibrium

Thermal Expansion

The Ideal Gas Law

Molecular Interpretation of Temperature

Temperature and Thermometers

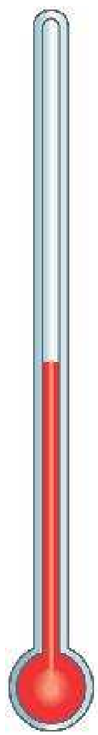
Temperature is a measure of how hot or cold something is.

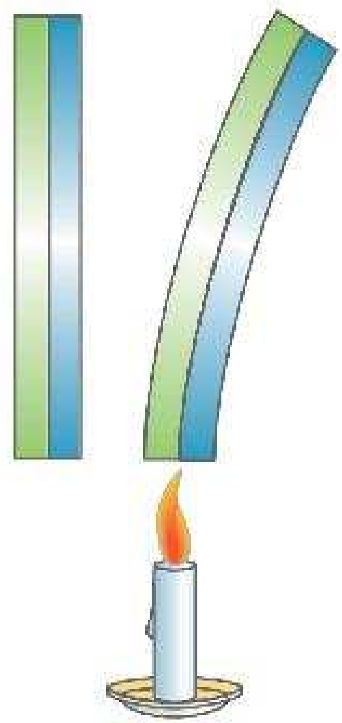
Thermometers are instruments designed to measure temperature. In order to do this, they take advantage of some property of matter that changes with temperature.

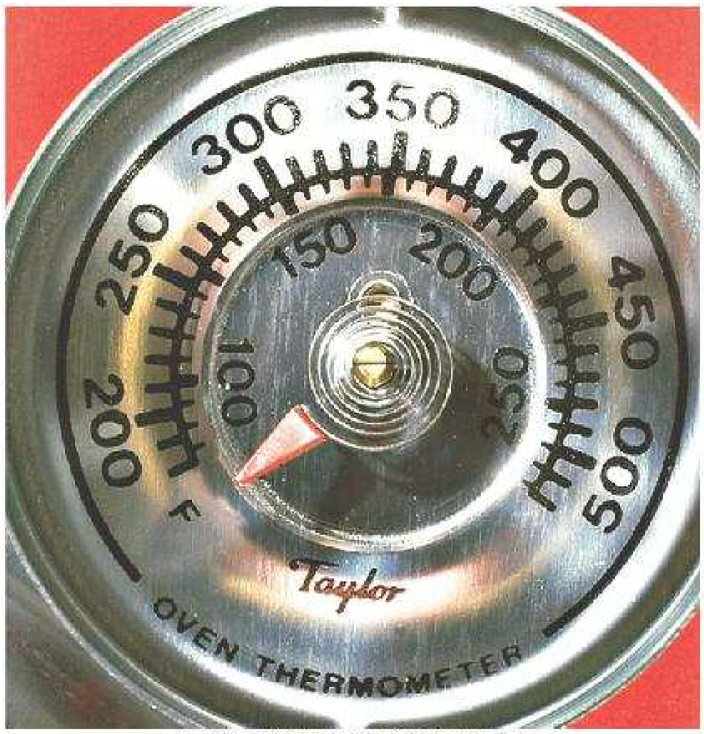
Most materials expand when heated.

Temperature and Thermometers

Common thermometers used today include the liquid-in-glass type and the bimetallic strip.







Temperature and Thermometers

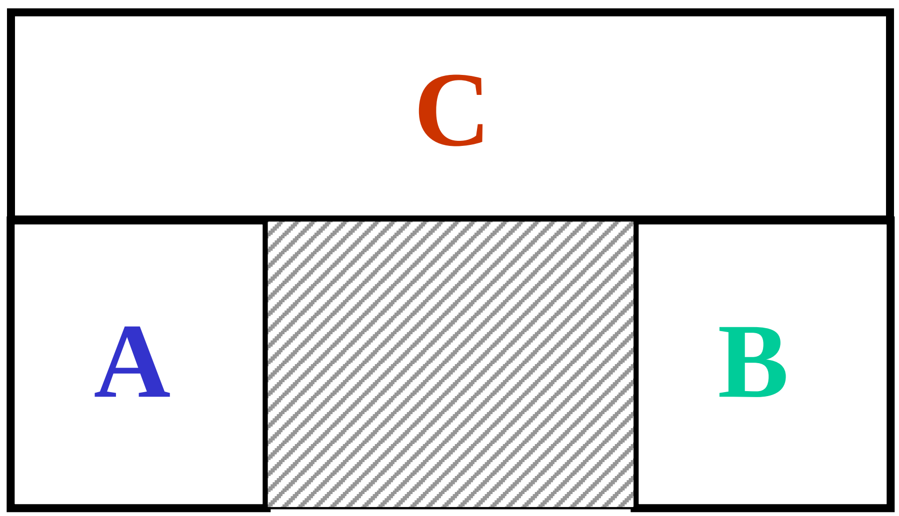
Temperature is generally measured using either the Kelvin, Celsius, or the Fahrenheit scale.

	Kelvin	Celsius	Fahrenheit
Boiling Point (H_2O)	373	100	212
Melting Point (H_2O)	273	0	32
Absolute Zero	0	-273	-459

Thermal Equilibrium

Two objects placed in thermal contact will eventually come to the same temperature. When they do, we say they are in thermal equilibrium.

The Zeroth Law of Thermodynamics

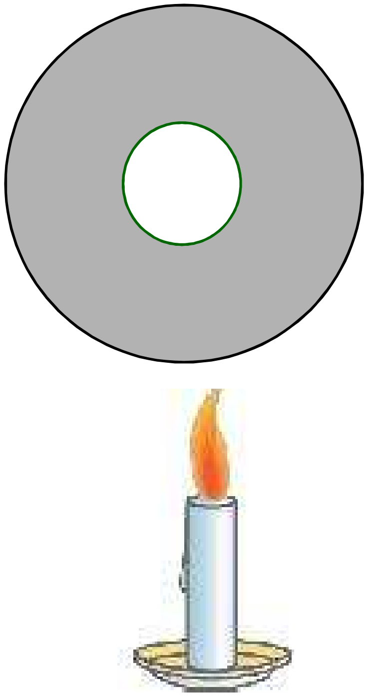


If A is in thermal equilibrium with C
and B is in thermal equilibrium with C
Then A and B are in thermal equilibrium.

Thermal Expansion

Expansion occurs when an object is heated.

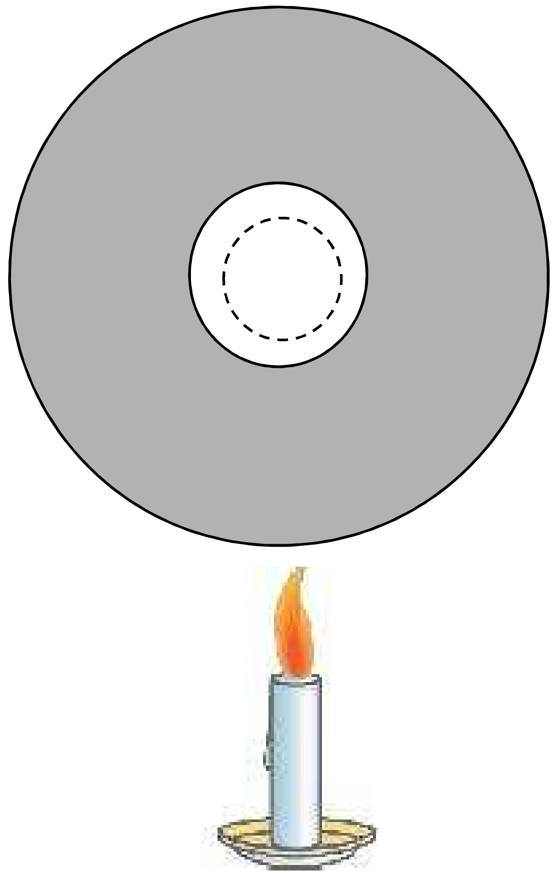
A steel washer is heated



Does the hole increase or decrease in size?

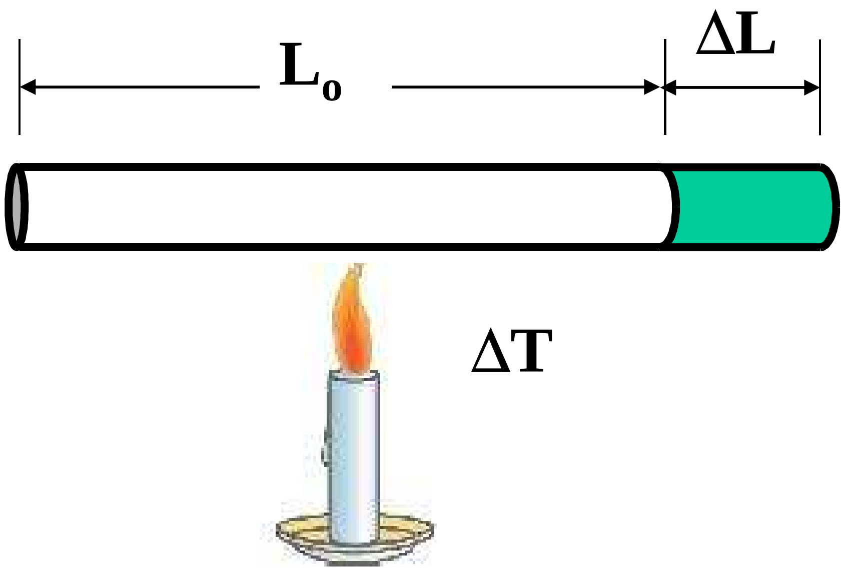
Thermal Expansion

When the washer is heated



The hole becomes larger

Thermal Expansion



$$\Delta L \propto L_o \Delta T$$

$$\Delta L = \alpha L_o \Delta T$$

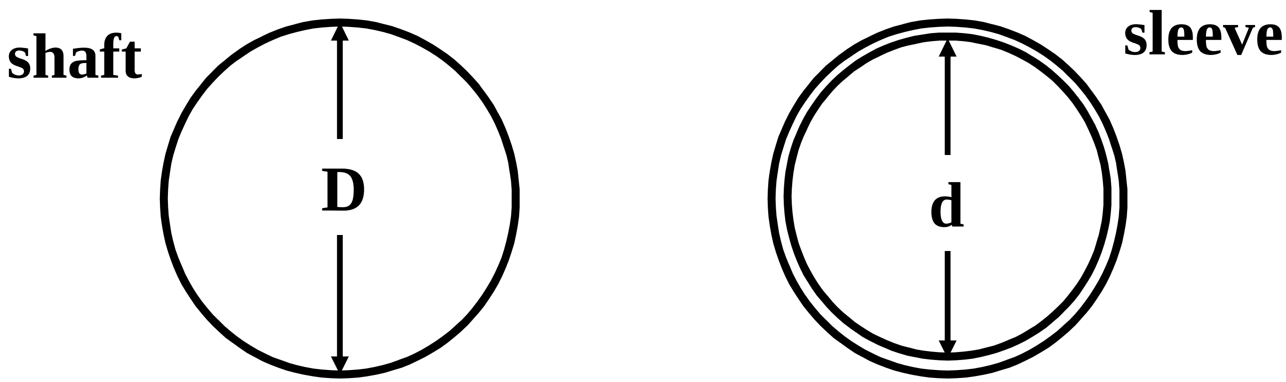
Coefficient of linear expansion

$$L - L_o = \alpha L_o \Delta T$$

$$L = L_o(1 + \alpha \Delta T)$$

Thermal Expansion

Expansion Problem



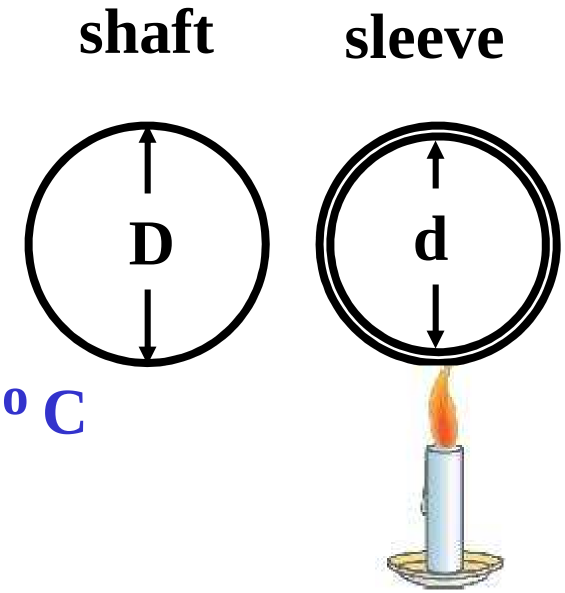
A cylindrical brass sleeve is to be shrunk-fitted over a brass shaft whose diameter is 3.212 cm at 0 ^oC. The diameter of the sleeve is 3.196 cm at 0 ^oC.

To what temperature must the sleeve be heated before it will slip over the shaft?

Thermal Expansion

Expansion Problem

To what temperature must the sleeve be heated before it will slip over the shaft?



$$\Delta L = D - d$$

$$\alpha = 19 \times 10^{-6}\ 1/^{o}C$$

$$D = 3.212\ cm$$

$$d = 3.196\ cm$$

$$T_i = 0^{o}C$$

$$\Delta L = \alpha L_i \Delta T$$

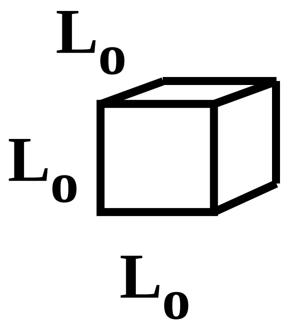
$$D - d = \alpha d (T_f - T_i)$$

$$D - d = \alpha d T_f$$

$$T_f = \frac{(D-d)}{\alpha d} = \frac{(3.212\ cm - 3.196\ cm)}{19 \times 10^{-6}\ 1/^{o}C\,(3.196\ cm)} = 263\ ^{o}C$$

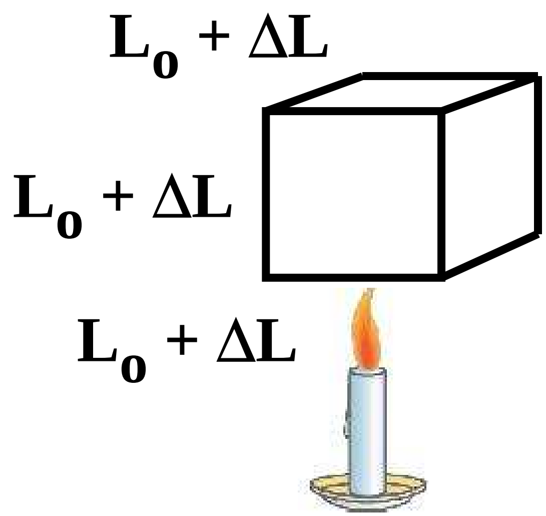
Thermal Expansion

Volume Expansion



Initial Volume

$$V_o = L_o^3$$



New Volume

$$V = (L_o + \Delta L)^3$$

$$V = L_o^3 + 3L_o^2\Delta L + \cancel{3L_o(\Delta L)^2} + \cancel{\Delta L^3}$$

$$\Delta V = V - V_o = 3L_o^2\Delta L$$

$$\Delta L = \alpha L_o \Delta T$$

$$\Delta V = 3L_o^2(\alpha L_o \Delta T)$$

$$\Delta V = 3\alpha L_o^3 \Delta T$$

$$\boxed{\Delta V = 3\alpha V_o \Delta T}$$

Thermal Expansion

$$\Delta V = 3\alpha V_o \Delta T$$

Coefficient of Volume Expansion

$$\beta = 3\alpha$$

$$\Delta V = \beta V_o \Delta T$$

$$V - V_o = \beta V_o \Delta T$$

$$V = V_o(1 + \beta\Delta T)$$

Thermal Expansion

Expansion Problem

An automobile fuel tank is filled to the brim with 45 L of gasoline at 10 ºC. Immediately afterward, the vehicle is parked in the Sun, where the temperature is 35 ºC. How much gasoline overflows from the tank as a result of expansion?

Overflow

Change in volume of the steel gas tank

Change in volume of the gasoline

$$\Delta V = \Delta V_G - \Delta V_S$$

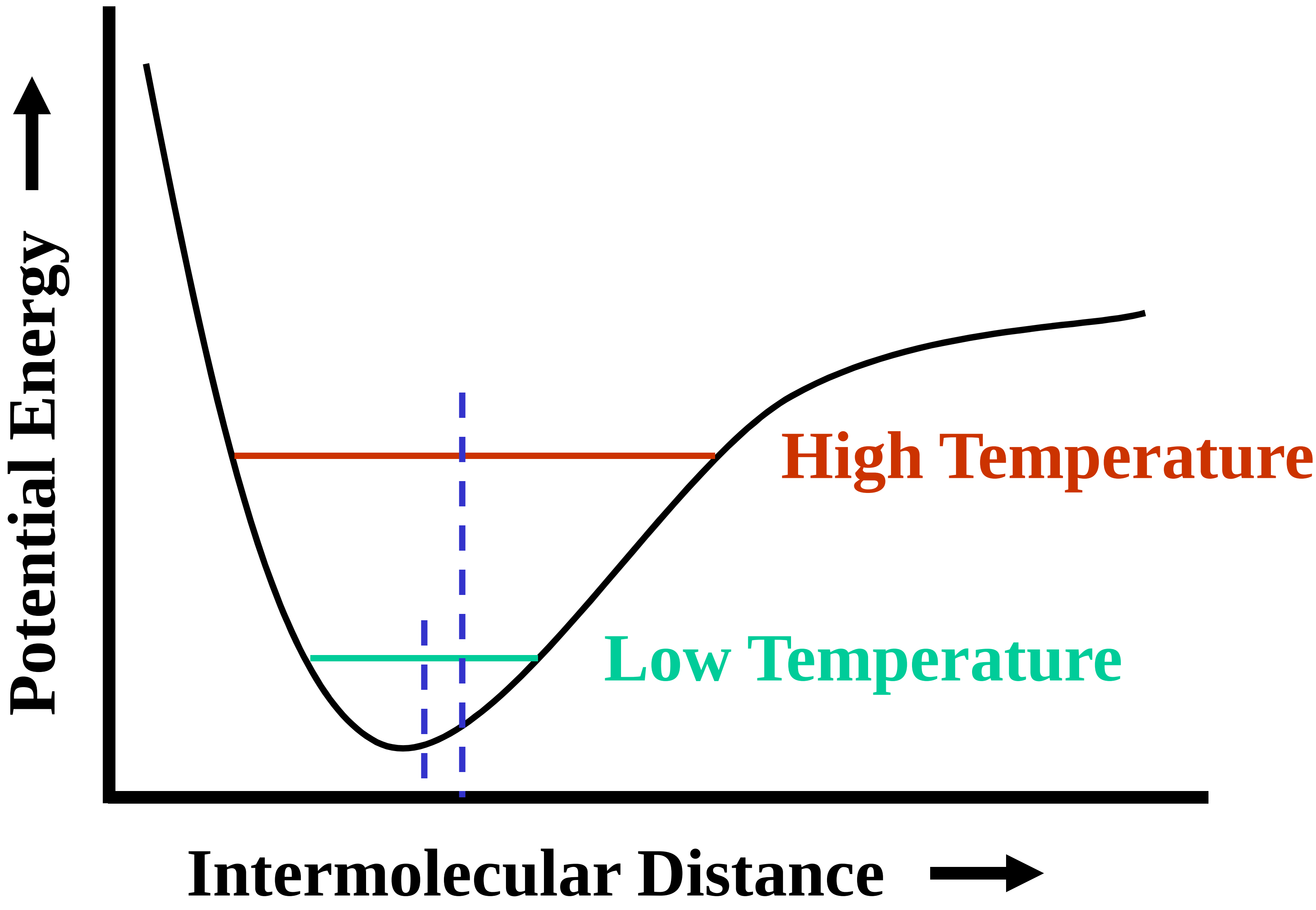
$$\Delta V = \beta_G V_o \Delta T - \beta_S V_o \Delta T$$

$$\Delta V = (\beta_G - \beta_S) V_o \Delta T$$

$$\Delta V = \left(9.6 \times 10^{-4} - 33 \times 10^{-6}\right) 45(25)$$

$$\boxed{\Delta V = 1.04\ \text{L}}$$

Thermal Expansion



The Ideal Gas Law

$$PV = nRT$$

Pressure (Pa)

Volume (m^3)

Absolute Temperature (K)

Gas Quantity (mol)

Gas Constant
(8.31 J/mol·K)
0.0821 (L·atm)/(mol·K)
1.99 cal/(mol·K)

The Ideal Gas Law

A mole (mol) is defined as the number of grams of a substance that is numerically equal to the molecular mass of the substance:

1 mol H_2 has a mass of 2 g
1 mol Ne has a mass of 20 g
1 mol CO_2 has a mass of 44 g

The number of moles in a certain mass of material:

$$n\,(\text{mol}) = \frac{\text{mass (grams)}}{\text{molecular mass (g/mol)}} \Rightarrow n = \frac{m}{M}$$

The number of moles in a certain number of particles:

$$n\,(\text{mol}) = \frac{\text{molecules (particles)}}{\text{Avogadro's number (particles/mol)}} \Rightarrow n = \frac{N}{N_A}$$

The Ideal Gas Law

$$PV = NkT$$

Pressure (Pa)

Volume (m^3)

Absolute Temperature (K)

Number of Molecules

Boltzmann's Constant (1.38×10^{-23} J/K)

The Ideal Gas Law

Boltzmann's Constant

$$PV = nRT = NkT$$

$$k = \frac{nR}{N}$$

Gas Constant

$$k = \frac{R}{N/n}$$

Avogadro's Number

The Ideal Gas Law

A gas is contained in an 8.0 x 10^{-3} m^3 vessel at 20 °C and a pressure of 9.0 x 10^5 N/m^2.

(a) Determine the number of moles of gas in the vessel.

$$PV = nRT$$

$$n = \frac{PV}{RT} = \frac{9.0\,x\,10^5\ N/m^2\left(8.0\,x\,10^{-3}\,m^2\right)}{8.31\,J/mol\cdot K\left(293\,K\right)}$$

$$\boxed{n = 3.0\,mol}$$

The Ideal Gas Law

A gas is contained in an 8.0 x 10^{-3} m^3 vessel at 20 oC and a pressure of 9.0 x 10^5 N/m^2.

$$n = 3.0\,\text{mol}$$

(b) How many molecules are in the vessel?

$$N = nN_A$$

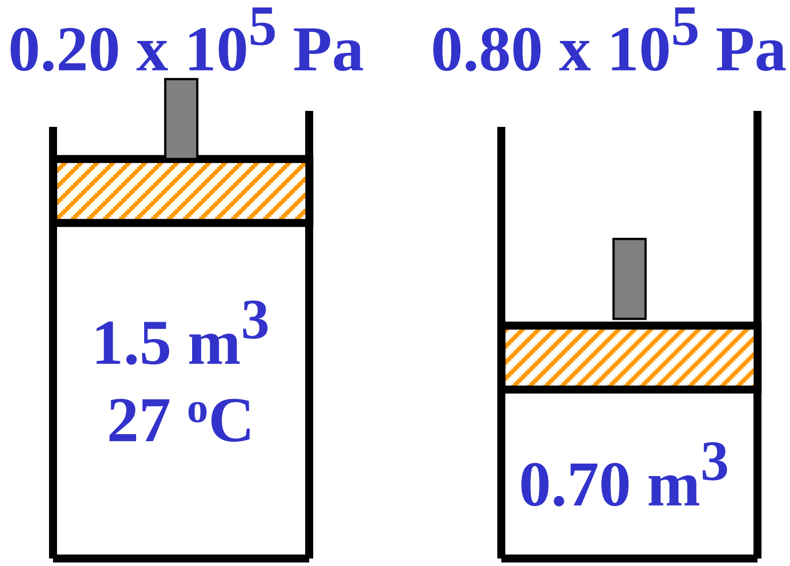
$$= 3.0\,\text{mol}\left(6.02\times 10^{23}\,\frac{\text{molecules}}{\text{mol}}\right)$$

$$\boxed{N = 1.8\times 10^{24}\,\text{molecules}}$$

The Ideal Gas Law

Problem

A cylinder with a moveable piston contains gas at a temperature of 27 ºC, a volume of 1.5 m^3, and an absolute pressure of 0.20 x 10^5 Pa.

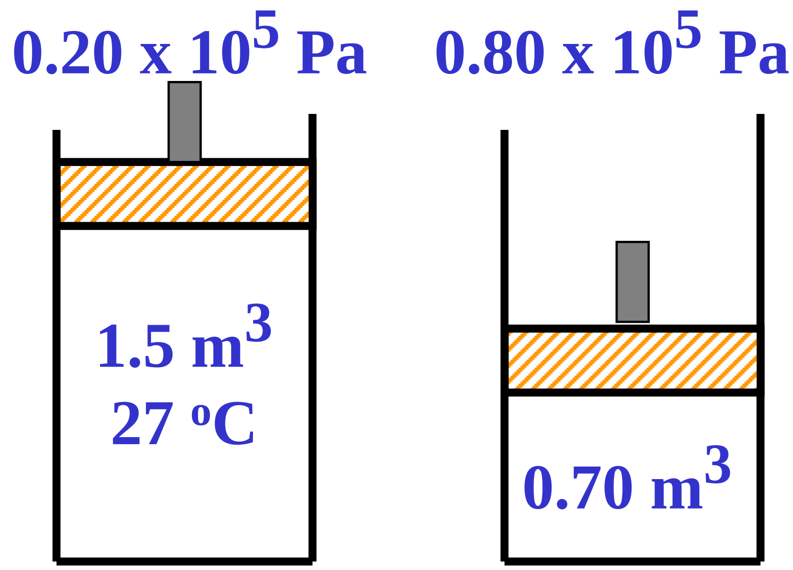


What will be its final temperature if the gas is compressed to 0.70 m^3 and the absolute pressure increases to 0.80 x 10^5 Pa?

The Ideal Gas Law

Problem

What will be its final temperature if the gas is compressed to 0.70 m^3 and the absolute pressure increases to 0.80 x 10^5 Pa?



$$PV = nRT \Rightarrow \frac{PV}{T} = nR = \text{constant}$$

$$\frac{P_f V_f}{T_f} = \frac{P_i V_i}{T_i}$$

$$T_f = T_i\left(\frac{P_f V_f}{P_i V_i}\right) = 300\,\text{K}\,\frac{\left(0.8 \times 10^5\ \text{Pa}\right)0.7\,\text{m}^3}{\left(0.2 \times 10^5\ \text{Pa}\right)1.5\,\text{m}^3} = 560\,\text{K}$$

$$-273$$

287 °C

Molecular Interpretation of Temperature

Assumptions of kinetic theory:

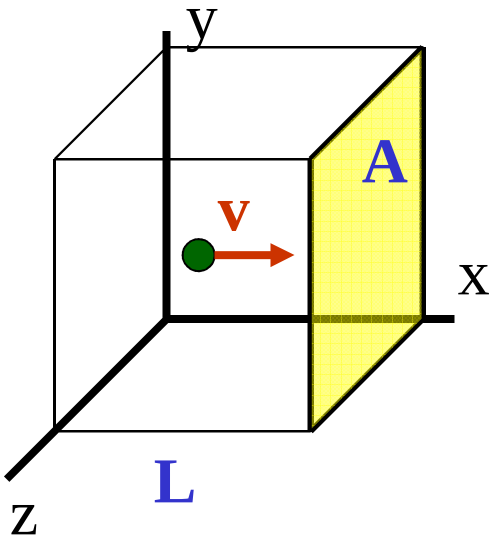
1) large number of molecules, moving in random directions with a variety of speeds

2) molecules are far apart, on average

3) molecules obey laws of classical mechanics and interact only when colliding

4) collisions are perfectly elastic

Molecular Interpretation of Temperature



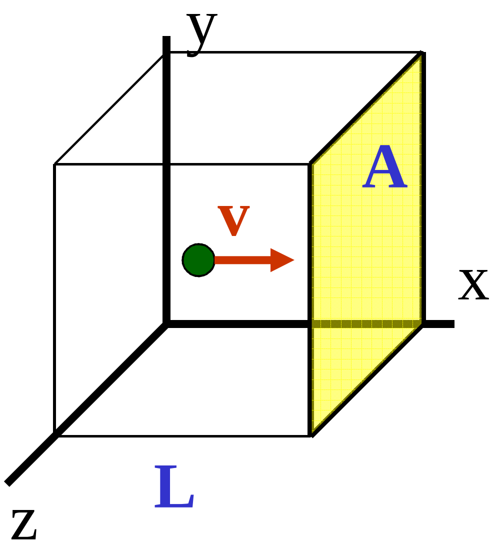
The force exerted on the wall by the collision of one molecule of mass **m** is

$$F = \frac{\Delta(mv)}{\Delta t} = \frac{2mv_x}{2L/v_x} = \frac{mv_x^2}{L}$$

Then the average force due to **N** molecules colliding with that wall is

$$\overline{F} = \frac{m}{L} N \overline{v_x^2}$$

Molecular Interpretation of Temperature



The averages of the squares of the speeds in all three directions are equal:

$$\overline{F} = \frac{1}{3}\left(\frac{mN\overline{v^2}}{L}\right)$$

$$\overline{F} = \frac{m}{L}N\overline{v_x^2}$$

So the pressure on the wall is:

$$P = \frac{\overline{F}}{A} = \frac{1}{3}\frac{Nm\overline{v^2}}{AL} = \frac{1}{3}\frac{Nm\overline{v^2}}{V}$$

Molecular Interpretation of Temperature

$$P = \frac{1}{3}\frac{Nm\overline{v^2}}{V}$$

Rewriting,

$$PV = \frac{2}{3}N\left(\frac{1}{2}m\overline{v^2}\right) = NkT$$

so

$$\frac{2}{3}\left(\frac{1}{2}m\overline{v^2}\right) = kT$$

$$\left(\overline{KE}\right) = \frac{1}{2}m\overline{v^2} = \frac{3}{2}kT$$

The average translational kinetic energy of the molecules in an ideal gas is directly proportional to the temperature of the gas.

Molecular Interpretation of Temperature

Molecular Kinetic Energy

Temperature is a measure of the average molecular kinetic energy.

$$\frac{m\overline{v^2}}{2}$$

$$\overline{KE} = \frac{3kT}{2}$$

$$\frac{m\overline{v^2}}{2} = \frac{3kT}{2}$$

$$v_{(rms)} = \sqrt{\frac{3kT}{m}}$$

Molecular Interpretation of Temperature

Problem

What is the total random kinetic energy of all the molecules in one mole of hydrogen at a temperature of 300 K.

Avogadro's number

Kinetic energy per molecule

$$K = N_A\left(\frac{3}{2}kT\right)$$

$$K = 6.02\,x\,10^{23}\ \text{molecules}\left(\frac{3}{2}\right)1.38\,x\,10^{-23}\ \frac{J}{K}\left(300\ K\right)$$

$$\boxed{K = 3740\ J}$$

Molecular Interpretation of Temperature

Problem

Calculate the rms speed of a Nitrogen molecule (N_2) when the temperature is 100 °C.

Mass of N_2 molecule:

$$m = \frac{28.0 \times 10^{-3}\ \text{kg/mole}}{6.02 \times 10^{23}\ \text{molecules/mole}} = 4.65 \times 10^{-26}\ \text{kg}$$

rms speed:

$$v_{(rms)} = \sqrt{\frac{3kT}{m}} = \sqrt{\frac{3\left(1.38 \times 10^{-23}\ \text{J/K}\right)373\ \text{K}}{4.65 \times 10^{-26}\ \text{kg}}}$$

$$v_{(rms)} = 576\ \text{m/s}$$

Molecular Interpretation of Temperature

Problem

If 2.0 mol of an ideal gas are confined to a 5.0 L vessel at a pressure of 8.0 x 10^5 Pa, what is the average kinetic energy of a gas molecule?

Temperature of the gas:

$$PV = nRT$$

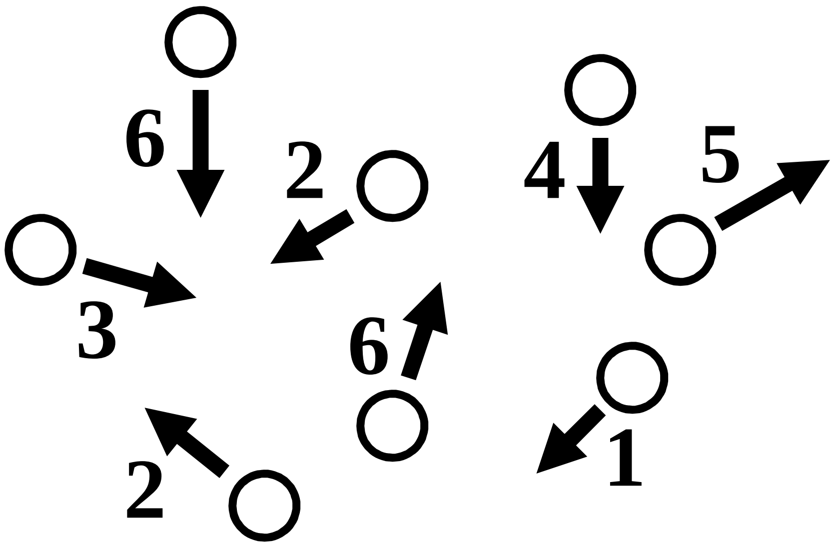
$$T = \frac{PV}{nR} = \frac{8.0 \times 10^5\ \text{Pa}\left(5.0 \times 10^{-3}\ \text{m}^3\right)}{2.0\ \text{mol}\left(8.31\ \frac{\text{J}}{\text{mol} \cdot \text{K}}\right)} = 244\ \text{K}$$

Kinetic energy:

$$K = \frac{3}{2}kT = \frac{3}{2}\left(1.38 \times 10^{-23}\ \frac{\text{J}}{\text{K}}\right)244\ \text{K} \quad \boxed{= 1.38 \times 10^{-23}\ \frac{\text{J}}{\text{molecule}}}$$

Molecular Interpretation of Temperature

Mean and rms Speed



Mean Speed:

$$= \frac{1+6+4+2+6+3+2+5}{8}$$

$$v_{(mean)} = 3.6\ \text{m/s}$$

rms Speed:

$$= \sqrt{\frac{(1)^2+(6)^2+(4)^2+(2)^2+(6)^2+(3)^2+(2)^2+(5)^2}{8}}$$

$$v_{(rms)} = 4.0\ \text{m/s}$$

Summary

Temperature is a measure of how hot or cold something is, and is measured by thermometers.

There are three temperature scales in use: Celsius, Fahrenheit, and Kelvin.

When heated, a solid will get longer by a fraction given by the coefficient of linear expansion.

The fractional change in volume of gases, liquids, and solids is given by the coefficient of volume expansion.

Summary

Ideal gas law: $PV = nRT$

One mole of a substance is the number of grams equal to the atomic or molecular mass.

Each mole contains Avogadro's number of atoms or molecules.

The average kinetic energy of molecules in a gas is proportional to the temperature:

$$\left(\overline{KE}\right) = \frac{1}{2} m \overline{v^2} = \frac{3}{2} kT$$


END
325
50
575
300