A.P. Physics
Chapter 12 Overview

Gravity and Orbits

Gravitational Potential Energy

Approximation near the surface: $U_G = mgh$

where h is a small distance above surface, with U at the surface defined as zero.

In general: $U_G = \frac{-GMm}{r}$

where r is the center-to-center distance between two objects. (A single object can't have potential energy.)

$$\vec{F} = \frac{-Gm_1m_2}{r^2}\hat{r}$$

- Gravitational force is a vector along the line between the centers of two objects.
- The universal gravitational constant, G, is the same everywhere, to our knowledge. (See Cavendish experiment.)
- The force is equal in magnitude between any two objects but opposite in direction.

$$U_G = \frac{-GMm}{r}$$

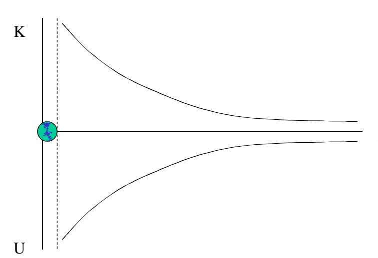
Gravitational potential energy is calculated relative to a central body (such as Earth). As r increases, the value of U becomes less negative, so the potential energy approaches zero as an object moves farther from the central body. This is also increase in potential energy (i.e., less and less negative) as an object moves farther from the central body.

*The gravitational force on an object is often called its weight, W.
*An object's weight near the surface of a body is *mg*.
*The value of g is dependent upon distance from center of the body, decreasing with increasing distance:

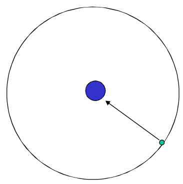
$$\vec{F}_G = m\vec{g}$$

$$g = \frac{GM}{r^2}$$

Consider an object from space that falls into Earth's gravitational field. It starts with a potential energy (relative to Earth) of zero and a kinetic energy of zero. Then as it falls toward Earth, its potential decreases (becomes more negative) as its kinetic energy increases.



Consider a satellite moving in a circular orbit:



The centripetal force is provided by the gravitational force, so the satellite's speed in orbit is:

$$\frac{mv^2}{r} = \frac{GMm}{r^2} \longrightarrow v_{orbit} = \sqrt{\frac{GM}{r}}$$

Note: escape velocity is $v_{escape} = \sqrt{\frac{2GM}{r}}$

For objects in orbit around a central body, there are a few things to remember:

1. As long as the object is in orbit, there must be a gravitational force on it (the centripetal force to keep it in orbit), so an object in orbit is not weightless.
2. Astronauts in the Space Shuttle, for example, feel weightless because they are in free fall. They actually have a *g* value of about 8.7 m/s^2.
3. The equations we have derived are for circular orbits, but most objects are in elliptical orbits. [Remember, a circle is an ellipse with an eccentricity of zero.]
4. The radius used in these equations is the <u>orbital radius</u>, which is the radius of the central body plus height above the surface.

The speed of the satellite in orbit is also:

$$v = \frac{2\pi r}{T}$$

Setting these equal to each other:

$$\frac{2\pi r}{T} = \sqrt{\frac{GM}{r}}$$

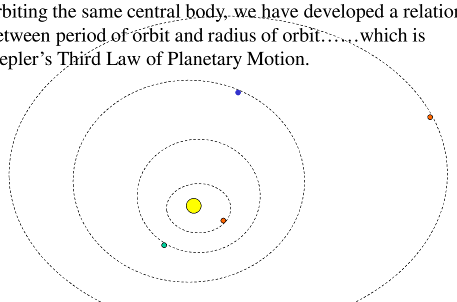
$$\frac{T^2}{r^3} = \frac{4\pi^2}{GM}$$

Let's look at this further.

<u>Geosynchronous</u> orbits are those in which the satellite has the same orbital period as Earth. If we substitute a period of 1 Earth day into previous equations, it's not difficult to find a specific orbital velocity and orbital radius for all geosynchronous orbits. Therefore, all geosynchronous satellites are moving in orbits at the same height above Earth's surface but will never collide, as long as they are "geostationary", i.e., moving the same direction as Earth turns so that they appear to "hover" over specific locations.

$$\frac{T^2}{r^3} = \frac{4\pi^2}{GM}$$

Since everything on the right side is constant for any satellite orbiting the same central body, we have developed a relationship between period of orbit and radius of orbit……which is Kepler's Third Law of Planetary Motion.



Geosynchronous Earth satellites orbit at 7 earth radii, or 1/9 the distance to the moon

Calculate the altitude and speed for an Earth satellite in geosynchronous orbit:

$$\frac{mv^2}{r} = \frac{GMm}{r^2} \quad and \quad v = \frac{2\pi r}{T}$$

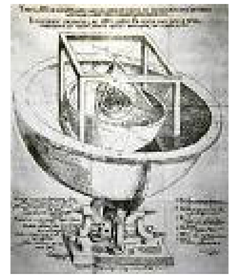
$$\frac{(2\pi r)}{86,400\,s} = \sqrt{\frac{GM}{r}} = \sqrt{\frac{(6.67E-11\,Nm/kg^2)(5.98E24\,kg)}{r}}$$

$$\frac{4\pi^2 r^2}{7.46E^9} = \frac{(6.67E-11\,Nm/kg^2)(5.98E24\,kg)}{r}$$

$$r = 4.22E7\,m$$

$$h = r - r_{earth} = 3.59E7\,m$$

This is 5.6 *Earth radii above the surface.*



Johannes Kepler and his planetary model.

If, for some reason, a satellite in circular orbit speeds up—and is moving too fast for the calculated speed for that orbital radius, it must either:

(a) move into a lower orbit, or

(b) "overshoot" the circular orbit, moving into an elliptical orbit with the central body as one focus.

The opposite happens if the satellite slows down slightly.

Sample Problem: Calculate the average orbital radius of Pluto, using what you know about the Solar System and Kepler's Laws and the fact that it takes Pluto 248 Earth years to make one orbit.

KEPLER'S LAWS

- The orbit of every planet is an ellipse, with the Sun at one focus.
- A line from the planet to the Sun sweeps equal areas in equal amounts of time.
- The square of the period of a planet in its orbit is directly proportional to the cube of its orbital radius:

$Gm_pM_s/R^2 = m_pv^2/R$ and $v = 2\pi R/T$

$\therefore Gm_pM_s/R^2 = m_p(2\pi R/T)^2/R$

and $T^2/R^3 = 4\pi^2/GM$

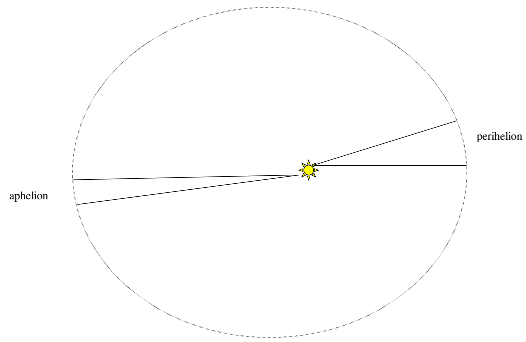
Note: This applies to any set of satellites orbiting the same central body.

Solution: Since Earth and Pluto both orbit the Sun, we can equate their orbital periods and orbital radii using Kepler's Third Law:

$T^2_E/R^3_E = T^2_P/R^3_P$

$(1\text{ year})^2/(1.5 \times 10^8\text{ km})^3 = (248\text{ years})^2/R^3_P$

$R_P = 5.9 \times 10^9$ km (which is almost 6 trillion meters!)

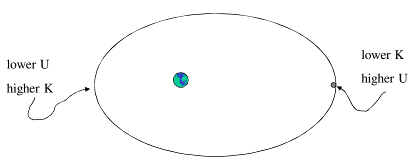


Note: Earth's perihelion occurs around Jan. 2 and aphelion around July 3 each year, so obviously proximity to the Sun is not what predicts our seasonal climate changes.

√ **Reminders:**

- * When calculating the gravitational force or gravitation potential energy, remember that the units in *G* demand that when substituting quantities into these equations all masses must be in kilograms and distances must be in meters.
- * The gravitational forces between any two masses are equal in magnitude and opposite in direction.
- * The gravitational force is always an attractive force.
- *The gravitational force follows an "inverse square law", i.e., doubling the distance between two objects cuts gravitational force in fourth, tripling the distance cuts in to one-ninth, etc.
- *The gravitational potential energy is directly proportional to distance from center of gravitational pull.
- *The mass, *m*, in the centripetal force formula is always the mass of the object in orbit.
- *Centripetal and gravitational forces are both negative, indicating force toward the center.
- *Kepler's Third Law relates objects that are orbiting the same central body.
- *The radius in the centripetal force and the gravitational force formulas is center-to-center, so if an altitude for a satellite in orbit is given, the radius of the orbited body must be added to the altitude to obtain orbital radius.

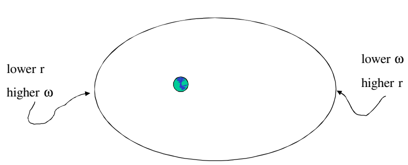
Conservation of Energy



$$E_T = U + K = \frac{-GMm}{r} + 1/2mv^2$$

As orbital radius decreases, the potential energy term becomes a greater negative number—or decreases. The kinetic energy increases, so the total energy remains constant throughout the orbit.

Conservation of Angular Momentum



$$\vec{L}_1 = \vec{L}_2$$
$$I_1\omega_1 = I_2\omega_2$$

1. As orbital radius decreases, the linear speed and angular speed increase, keeping angular momentum constant throughout the orbit..
2. Angular momentum for a single object is: **L** = mrv (or **L** = m **r × v**)