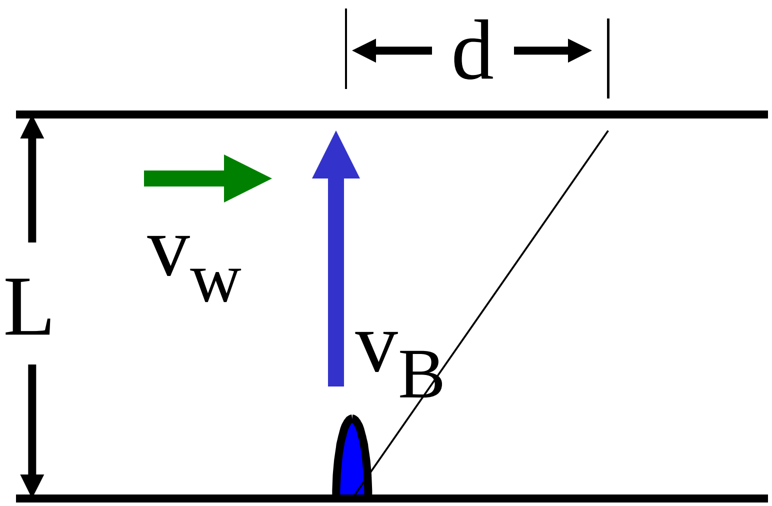
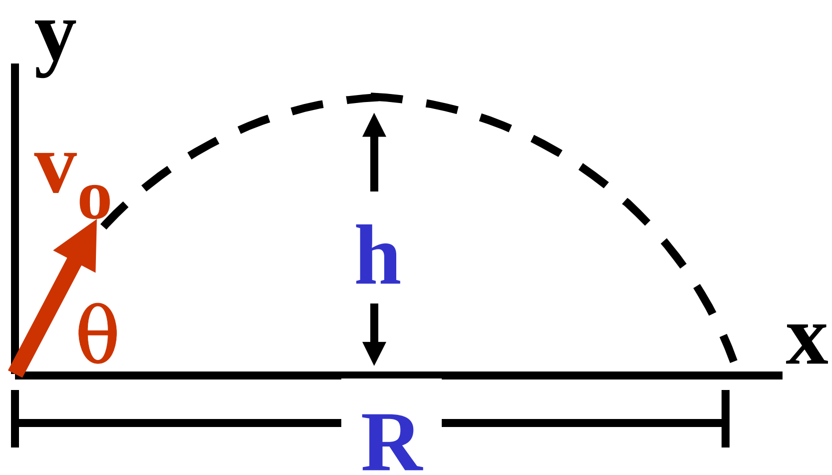
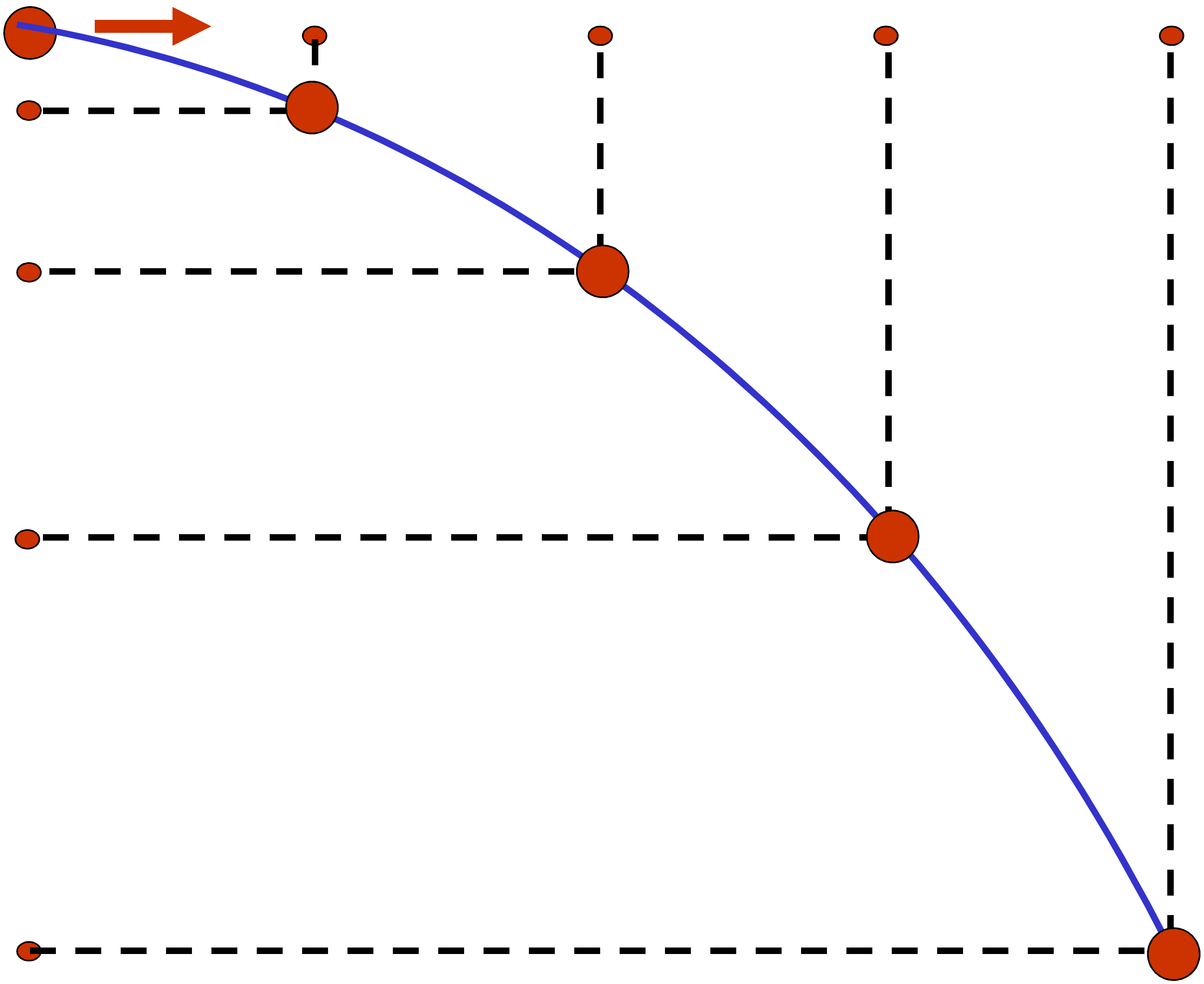
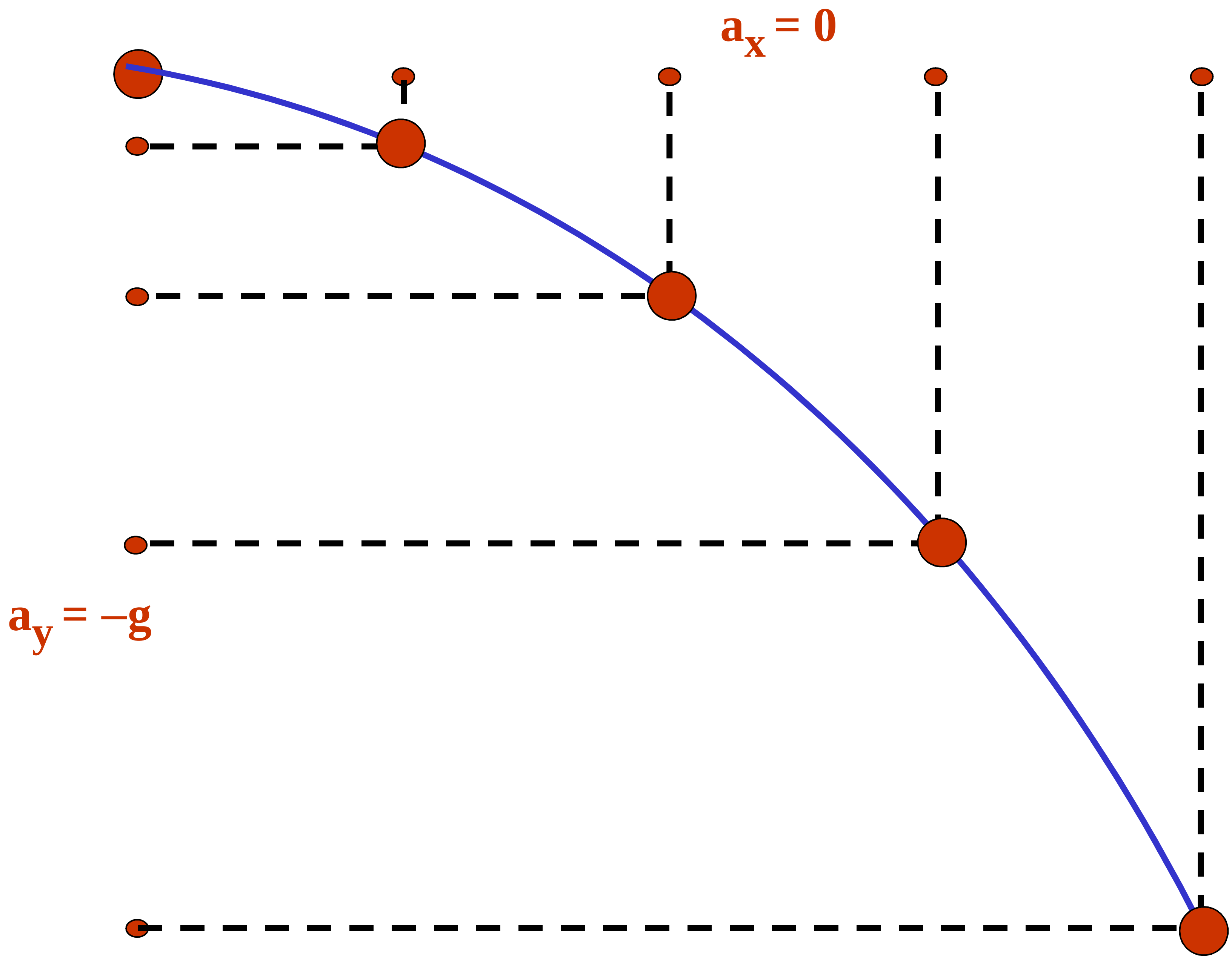
Projectile Motion And Relative Velocity

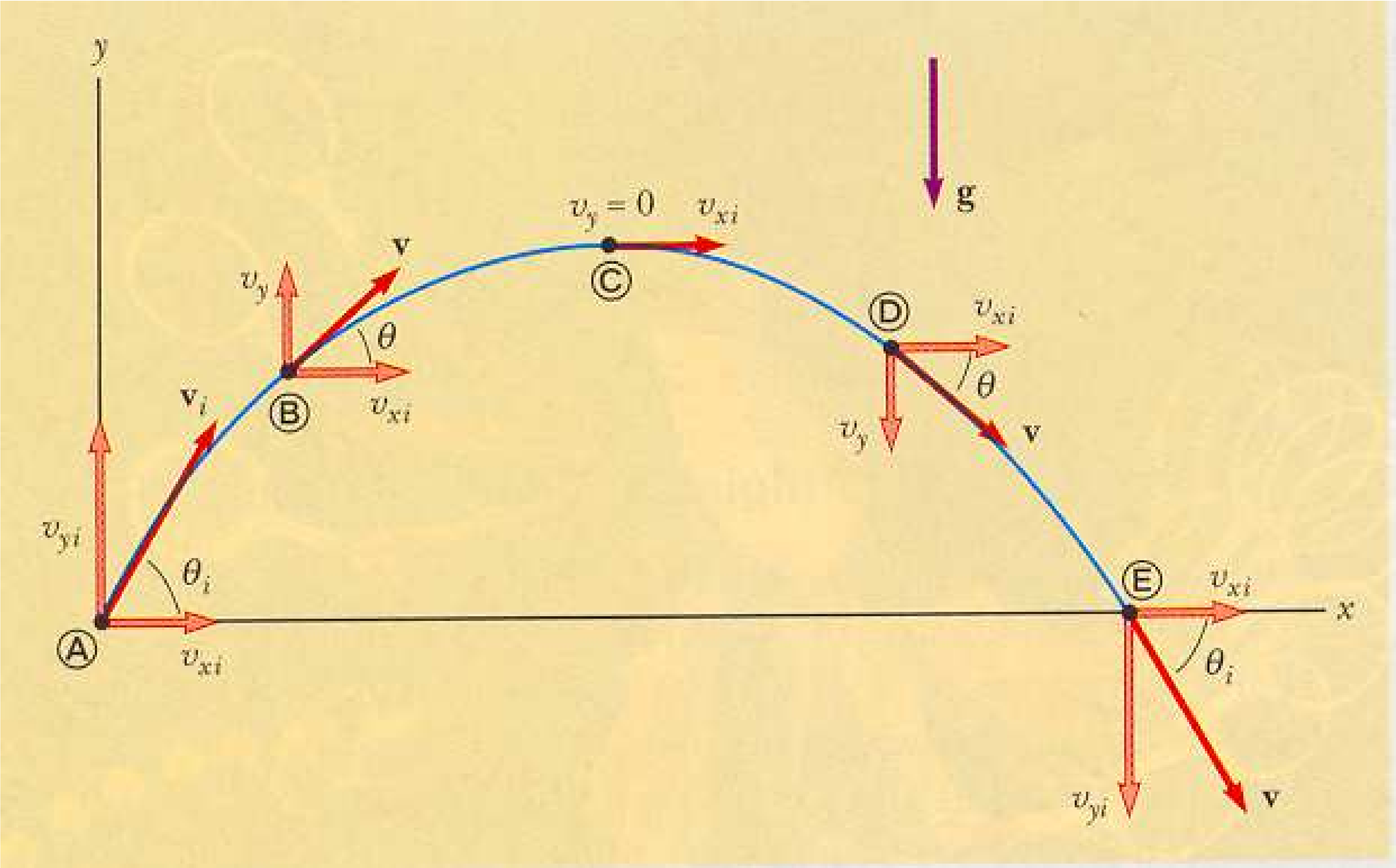




Physics 2053
Lecture Notes

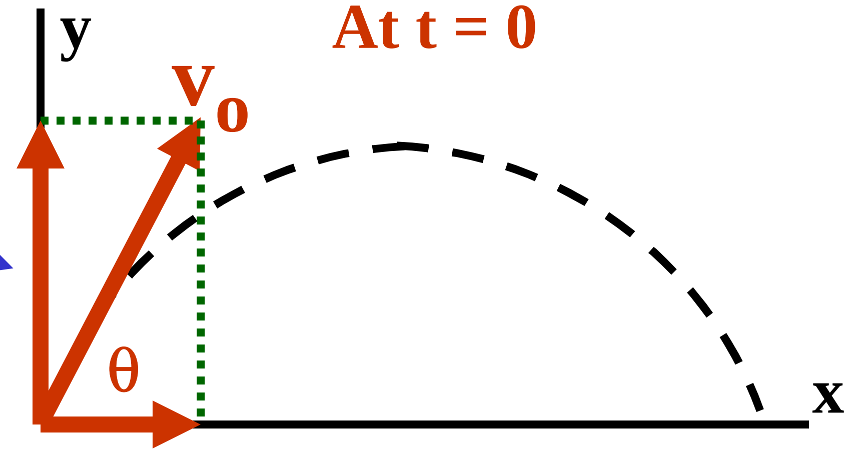



$a_x = 0$
$a_y = -g$


y
g
$v_y = 0$
v_{xi}
v
C
v_y
D
v_{xi}
θ
θ
v_i
B
v_{xi}
v_y
v
v_{yi}
$θ_i$
E
v_{xi}
x
A
v_{xi}
$θ_i$
v_{yi}
v

Horizontal Velocity and Vertical Velocity

$$v_{o_y} = v_o \sin(\theta)$$



<u>Constant acceleration</u>

$$v = v_o + at$$

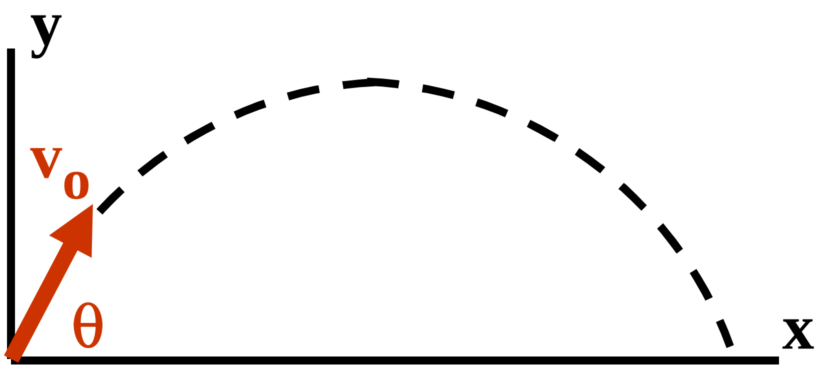
$$v_y = v_{o_y} + (-g)t$$

$$v_y = v_o \sin(\theta) - gt \qquad \text{Eq 2}$$

<u>Constant velocity</u>

$$v_x = v_o \cos(\theta) \qquad \text{Eq 1}$$

Horizontal Displacement



Constant velocity

$$v = \frac{\Delta x}{\Delta t}$$

$$v_x = v_o \cos(\theta) \quad \text{Eq. 1}$$

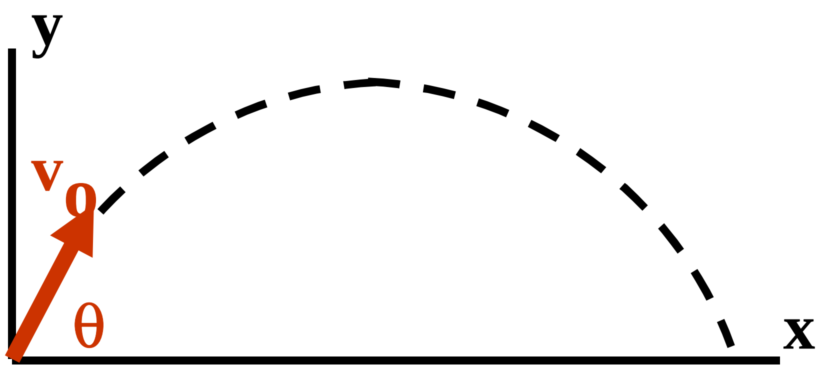
$$v_x = \frac{x}{t}$$

Sub Eq 1

$$v_o \cos\theta = \frac{x}{t}$$

$$x = v_o \cos(\theta)\, t \quad \text{Eq 3}$$

Vertical Displacement



Constant acceleration

$$x = x_o + v_o t + \frac{at^2}{2}$$

$$v_{o_y} = v_o \sin(\theta)$$

$$y = (0) + v_{o_y} t + \frac{(-g)t^2}{2}$$

$$y = v_o \sin\theta + \frac{(-g)t^2}{2}$$

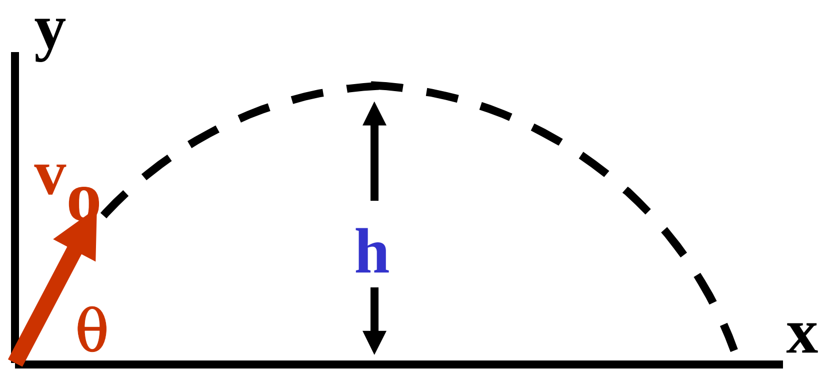
$$y = v_o \sin(\theta)\, t - \frac{gt^2}{2}$$

Eq 4

Vertical Position as a Function of Horizontal Displacement

$$x = v_o \cos(\theta) t \quad \text{Eq. 3}$$

$$y = v_o \sin(\theta) t - \frac{gt^2}{2} \quad \text{Eq. 4}$$



Solve
Eq 3 for t

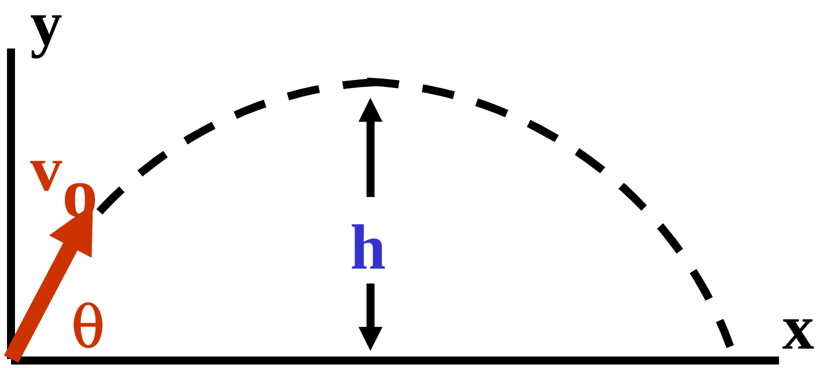
$$t = \frac{x}{v_o \cos(\theta)}$$

Sub into Eq 4

$$y = v_o \left(\frac{x}{v_o \cos(\theta)} \right) \sin(\theta) - \tfrac{1}{2} g \left(\frac{x}{v_o \cos(\theta)} \right)^2$$

$$\boxed{y = x \tan(\theta) - \frac{gx^2}{2v_o^2 \cos^2(\theta)}} \quad \text{Eq 5}$$

Maximum Height



$$y = v_i \sin(\theta) t - \frac{gt^2}{2} \quad \text{Eq. 4}$$

$$v_y = v_i \sin(\theta) - gt \quad \text{Eq. 2}$$

Sub into Eq 4

At the maximum height ($v_y = 0$)

$$v_y = v_i \sin(\theta) - gt = 0$$

$$y = v_i \sin(\theta)\left(\frac{v_i \sin(\theta)}{g}\right) - \frac{g}{2}\left(\frac{v_i \sin(\theta)}{g}\right)^2$$

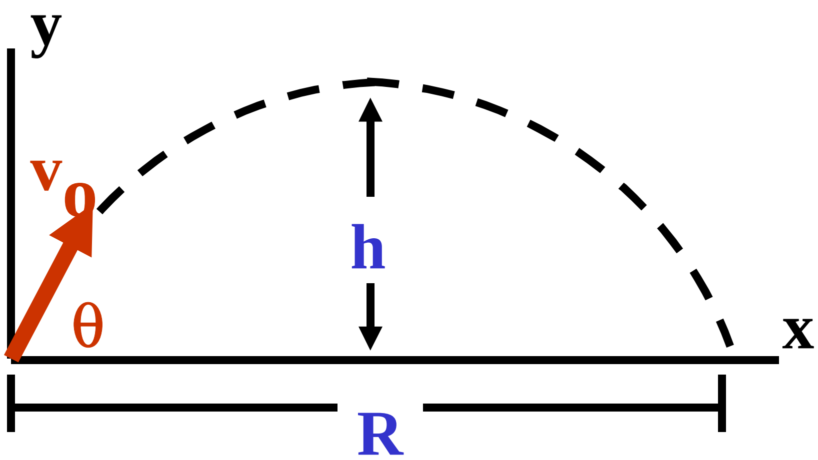
Eq 6

$$t_{top} = \frac{v_i \sin(\theta)}{g}$$

Eq 7

$$y_{max} = \frac{v_i^2 \sin^2(\theta)}{2g}$$

Range



$$v_x = v_o \cos(\theta) \quad \text{Eq. 1}$$

$$t_{top} = \frac{v_o \sin(\theta)}{g} \quad \text{Eq.6}$$

$$t_{air} = 2t_{top} = \frac{2v_o \sin(\theta)}{g}$$

$$R = v_x t_{air}$$

$$R = v_o \cos(\theta)\left(\frac{2v_o \sin(\theta)}{g}\right)$$

$$\boxed{R = \frac{v_o^2 \sin(2\theta)}{g}} \quad \text{Eq 8}$$

Equations

Horizontal Velocity

$$v_x = v_o \cos(\theta)$$

Eq 1

Vertical Velocity

$$v_y = v_o \sin(\theta) - gt$$

Eq 2

Horizontal Displacement

$$x = v_o \cos(\theta) t$$

Eq 3

Vertical Displacement

$$y = v_o \sin(\theta) t - \frac{gt^2}{2}$$

Eq 4

Equations

Vertical Position

$$y = x\tan(\theta) - \frac{gx^2}{2v_o^2\cos^2(\theta)}$$

Eq 5

Time to the Top

$$t_{top} = \frac{v_o\sin(\theta)}{g}$$

Eq 6

Maximum Height

$$y = h = \frac{v_o^2\sin^2(\theta)}{2g}$$

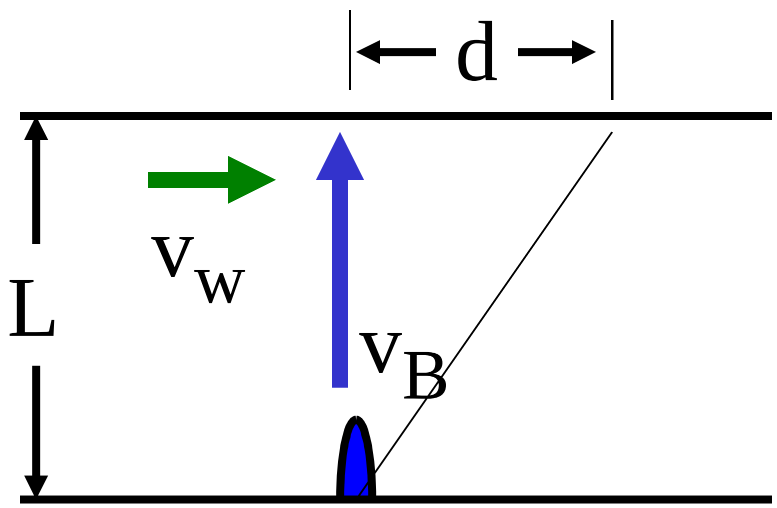
Eq 7

Range

$$R = \frac{v_o^2\sin(2\theta)}{g}$$

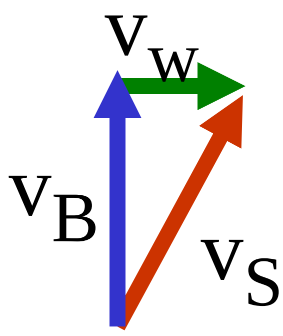
Eq 8

Relative Velocity



$$\frac{d}{L} = \frac{v_W}{v_B}$$

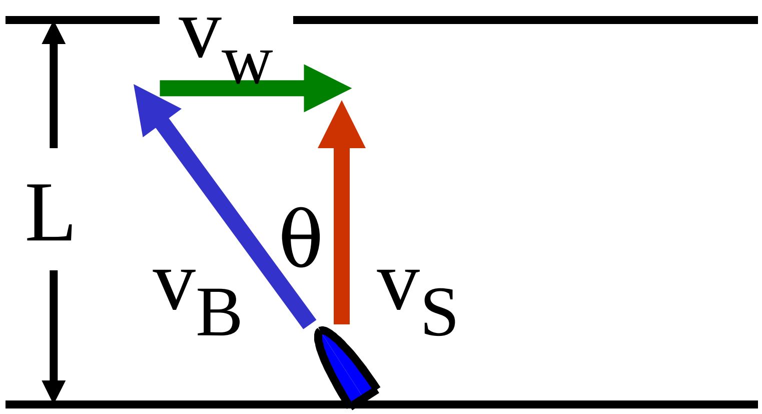
$$d = L\left(\frac{v_W}{v_B}\right)$$



$$v_S = \sqrt{v_W^2 + v_B^2}$$

Relative Velocity

Time required to go directly across.



$$\theta = \sin^{-1}\left(\frac{v_W}{v_B}\right)$$

$$v_s = \frac{L}{t}$$

$$v_B^2 = v_W^2 + v_S^2$$

$$v_S = \sqrt{v_B^2 - v_W^2}$$

$$t = \frac{L}{v_s} = \boxed{\frac{L}{\sqrt{v_B^2 - v_W^2}}}$$


END
325
575
50
300